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A STUDY OF ZERO-OUT AUCTIONS: EXPERIMENTAL ANALYSIS OF A PROCESS OF ALLOCATING PRIVATE RIGHTS TO THE USE OF PUBLIC PROPERTY

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ABSTRACT

The study examines a proposal to auction rights to land at a major airport and return the auction revenues to the winners. Experiments with such auctions are reported. New econometric models of the process are developed and evaluated.

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In 1984 the Port Authority of New York (the Port) issued a request for proposals for a study of methods to allocate the right to use the major New York airports. The Port was considering a special type of auction process which seemed to meet the Port's needs. The Port wanted to know the details of the process should it be implemented and how it would perform once in place. The contract was let and the consulting engineers (FTA 1985) delivered a recommendation. The research reported in this study has grown from questions motivated by the consultant's recommendations. National politics has intervened to prevent the Port from a continued study or to take action on the recommendation.

The issues are of interest to a broad audience of economists. The Port's problem is not unique. Several airports face similar problems and the solution adopted by the Port would have implications for the decisions of other airports and the future of the air transportation industry. More importantly, the problem faced by the Port is of a general type faced by certain public agencies that have been granted the right to allocate the private use of public resources. A study of the Port's problems has theoretical implications for several large segments of government at all levels.

This paper reports the results of experiments with a zero-out auction process considered by the Port. No other source of data about how the process might work exists. The type of model that should be applied or even the principles that underlie models of how the process might work in practice are controversial. The research in this paper rests on the assumption that a scientific understanding of what goes on in very simple cases that we are able to study will lead to better predictions about what will happen in the complex cases, like the Port's problem.

The paper is written to reflect a process of learning about the phenomena as the research progressed. First the Port's problem and the experimental setting are outlined in Sections I and II. A model that has proved useful in other market contexts, the static Nash equilibrium, is developed in Section III. Initial experiments answered some of the Port's questions but the obvious presence of dynamic adjustments motivated a second set of experiments and an extension of theory beyond static Nash. The data from two sets of experiments are presented in Section IV. Section V begins with the traditional approach to modeling dynamics as a series of equilibria. The section ends with an alternative approach and proposes two models that explain the dynamics in terms of individual optimization without system consistency constraints.

Historically, experimentalists have been hampered by an inability to deal with data generated by equilibrating markets. Theories about the dynamics of experimental markets are almost nonexistent. By going beyond the analysis of the static model, the paper is a first step in approaching this more general class of problems in a manner that is econometrically tractable. In particular, we shall propose some relatively simple theoretical and econometric dynamic models which will be evaluated on the basis of their explanatory and predictive power of the dynamics observed in the experiment.

SECTION I. THE PROPOSED POLICY

A. *The Port's Allocation Problem*

The Port Authority of New York operates three major airports (La Guardia, John F. Kennedy, and Newark). Growth of air traffic since deregulation has been such that further growth is prevented by the physical facilities and environmental, safety, and political considerations. More carriers wish to use the facility than can be accommodated. The Port would like to have an allocation process that is economically efficient in the sense that the most efficient carriers get the use of the Port's resources and that meets an additional distributional criterion that all of the surplus accrue to the users.

The problem is not new to governmental agencies. The private use of scarce public resources is frequently associated with the problem. Hunting permits are sometimes allocated randomly to applicants. Permits to access remote or natural areas and campsites are issued on a first come, fi served basis. Private permits to white water rivers are sometimes allocated by lottery. In all cases the resale of the permit is prohibited and great effort is expended to make sure that the user is the same person that was awarded the permit. The general allocation problem is to get the permits to the people who value them in use while simultaneously granting the user the full consumer's surplus. A preference revelation problem of substantial magnitude is involved. A satisfactory solution might not exist even in theory.

Clearly auction processes would do the job of getting the rights to land in the hands of the most efficient carriers, but the Port cannot take the money. Random allocation with an aftermarket would solve part of the problem but such a process has little chance of surviving the politics. The Port does not want to grandfather the rights to the resources and permit buying and selling. That process also faces political problems. So, the Port wanted to explore something new.

B. *The Zero-Out Auction*

The zero-out process recommended to the Port is simple in concept. The right to take off during a particular period of each day was defined as a "slot." Other constrained resources such as parking, gate space, baggage facilities, counter space, etc. are not addressed in this initial study. The process involves the periodic auction of slots which are to be fixed in number. Each hour of the day constitutes a different auction for the right to take off one time during that hour every day for the next year. Carriers desiring slots would tender a separate bid for each slot desired during a given hour. Available slots would be allocated to the highest bidders who would pay the amount of the bid.

The Port would not keep the funds (aside from the cost of administering the process). Instead the revenues would be distributed back to the carriers that operated at the airport by a function of the number of passengers enplaned in New York during the period. Specifically, suppose the carrier holding a slot at time t enplaned x passengers on that flight during the twelve-month period. Suppose further that a total of y passengers were enplaned at the airport over the twelve-month period. If the revenue generated by the auction is R, then $(x/y) \cdot R$ is rebated to the carrier at the end of the period.

The policy would permit an aftermarket in slots. Of course the rebate formula must be complicated by a process of accounting for the fact that slots might change hands. The lease of slots, the nonuse of slots, bankruptcy, etc. would be anticipated by the policy. However, these complications will not be considered in the analysis below.

SECTION II. THE EXPERIMENTAL APPROACH

A. *The Research Strategy*

While many issues are of interest to the Port,[1] the focus of this report is only on one dimension of the problem related to the auction process itself. Will the process involve some sort of inflation of bids over the values which would occur if a regular auction were used as opposed to the zero-out auction? How much inflation would occur? Is the process efficient? What type of model would help answer those questions and what data would need to be collected in order to use the model? What type of confidence should be placed in the model? The other questions, such as fares or the implications of returning funds based on enplaned passengers, will not be addressed.

If the bids inflated sufficiently, the whole process could become a political embarrassment of major proportions. The Port could be ridiculed in the press. Inflation might have implications for efficiency. For example, small airlines might be excluded because they could not obtain financing for their bids. Even without the other dimensions of the problem, inflation is difficult.

The research strategy is to begin with a simple case to see if we are able to provide satisfactory answers there. Tools that cannot accurately explain what is seen in a simple setting would have no real claim to reliability when applied to the Port's problem. The purpose of this paper is to examine only a simple case as a test of basic ideas. If the tools predict well in the simple case, then the policy strategy would be to undertake additional studies which would increasingly complicate the study case until either we are unable to provide an answer or the Port would be prepared to make decisions based on answers that are available. Thus in this report we analyze a very simple case that isolates some of the central issues.

B. *Experimental Design*

The behavior of the process was studied under two sets of parameters that will be referenced as parameter set 1 and parameter set 2. Three experiments were conducted with each set. Aside from the parameters, which will be given below, the two sets of experiments were procedurally identical. Twelve subjects recruited from the California Institute of Technology were randomly divided into two groups, Market 1 and Market 2, with six buyers in each market. Market 1 can be interpreted as one time of day such as morning and Market 2 a different time such as afternoon.

Each subject i was assigned a redemption value V_i for a single unit and each subject was assigned a capacity number X_i. The value V_i can be interpreted as the profit that a carrier gains from the operation of a slot without considering the price paid for the slot. Of course, because of the single unit valuation, each subject wanted only a single unit, or slot, in the airport jargon. A total of four units (slots) were sold in each market. The capacity numbers can be interpreted as the number of passengers the carrier would enplane in New York if the carrier acquired the slot. The capacity numbers were common knowledge, but the redemption values were kept as private information. Each subject was also assigned a capital payment which was paid independently of his/her decisions. The capital payments were used to keep earnings above some minimum level.

The bidding procedure was a sealed-bid discriminative auction. So, in each market the four units were sold to the four highest bidders in that market. The winners received their redemption values plus a proportion of the revenue rebated from both markets. The proportion rebated is given by the ratio of the subject's capacity X_i to the sum of capacities of the winners in both markets. Those who do not win in the auction received only the capital payment assigned to them. After the instructions (see Appendix B) were read, a small test was given to make sure that each subject understood the details of the procedure. After every period the highest bid, the lowest accepted bid, and the highest rejected bid in each market, the winners, and the amount of each rebate were announced.

The two different parameter sets are in Table 1. The differences in the two sets reflect an attempt to change the predictions of the Nash model. The experimental results from parameter set 1 were known when the parameter set 2 was developed. The second set thus reflects an attempt to test aspects of theory for which set 1 proved inappropriate.

SECTION III. A STATIC EQUILIBRIUM MODEL

The complete information static Nash equilibrium model has a long and reasonably successful history in applied work. Experimental markets are regularly observed converging to such equilibria in dynamic settings even when the underlying informational and behavioral assumptions of this equilibrium concept are not exactly satisfied in the experimental design (Plott 1982; Smith 1982). Thus, we first characterize the Nash equilibria (NE) in a one-shot game model which reflects all the simplifying and special conditions of the experiment. Although slight generalizations of the assumptions are consistent with the existence of a Nash equilibrium, we do not pursue such generalizations, as our aim is the analysis of the particular experiments.

Figures 1 and 2 contain a diagram of the Nash equilibria in parameter set 1 and parameter set 2 respectively. The vertical axis is the magnitude of the winning bid in market 1 and the horizontal axis is the winning bid in market 2. The diamond shaped figures are the sets of Nash equilibrium bids for the parameters. The lines A and B bounding the base of the diamonds are the pairs of bids that give expected zero profit after rebate to agents five and six of market 1 and agents eleven and twelve in market 2. At the static Nash equilibria these are the extramarginal (first excluded) agents in the two markets. The upper bounds of the rhomboid, lines C and D, are the pairs of bids that give zero profi to agents four and ten who are the marginal (last included) agents in markets 1 and markets 2 respectively.

Table 1: Parameters

	Exp/Market	Subject	Value, V_i	Capacity, X_i
Parameter Set 1	Exp 1,2,3 Market 1	1	100	20
		2	100	20
		3	75	15
		4	75	15
		5	50	15
		6	50	15
	Exp 1,2,3 Market 2	7	100	15
		8	100	15
		9	50	10
		10	50	10
		11	25	10
		12	25	10
Parameter Set 2	Exp 4,5,6 Market 1	1	100	200
		2	100	200
		3	75	200
		4	75	100
		5	50	100
		6	50	100
	Exp 4,5,6 Market 2	7	75	100
		8	75	100
		9	50	100
		10	50	50
		11	25	50
		12	25	50

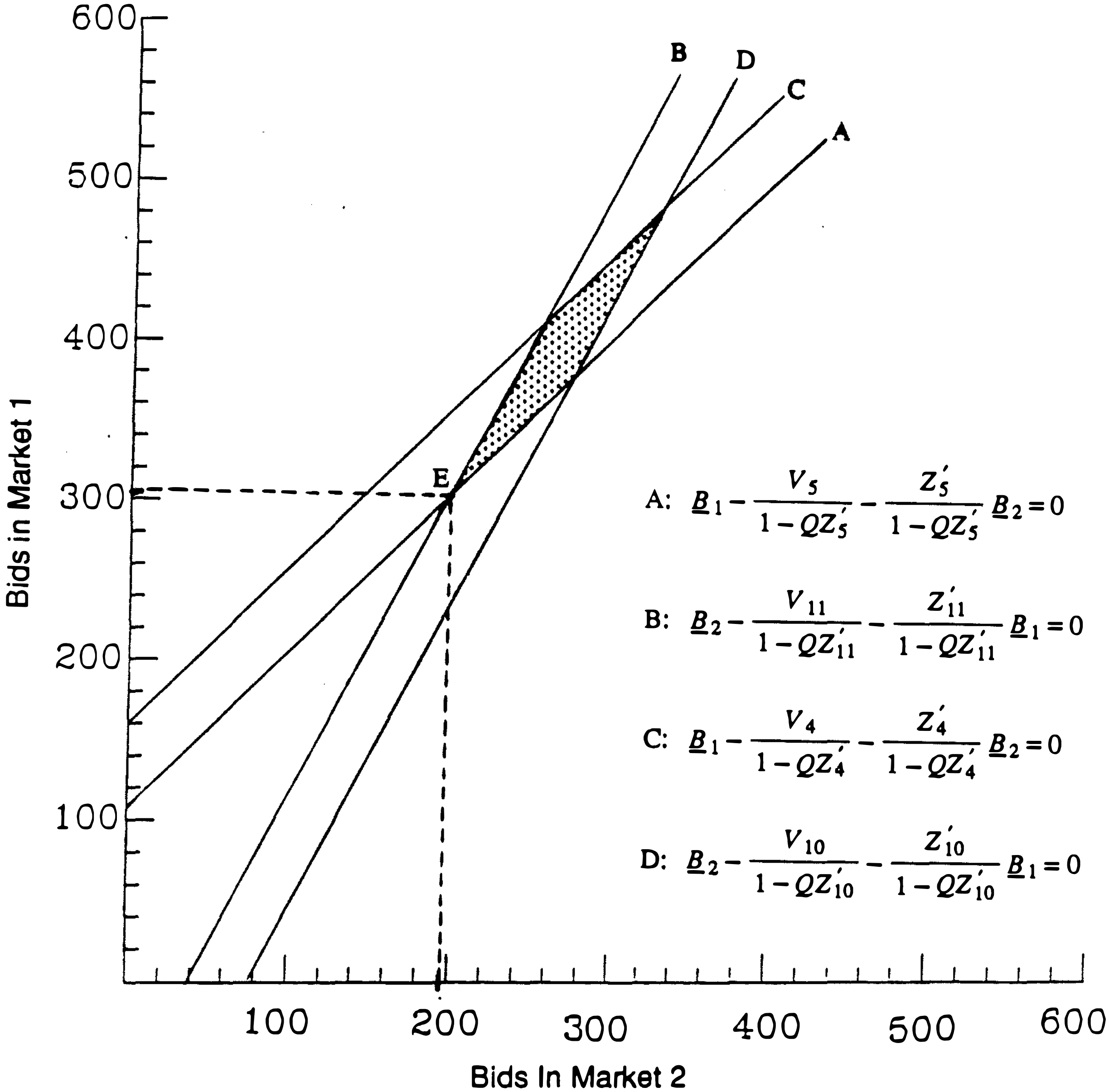


FIGURE 1: Nash Equilibria Parameter Set 1

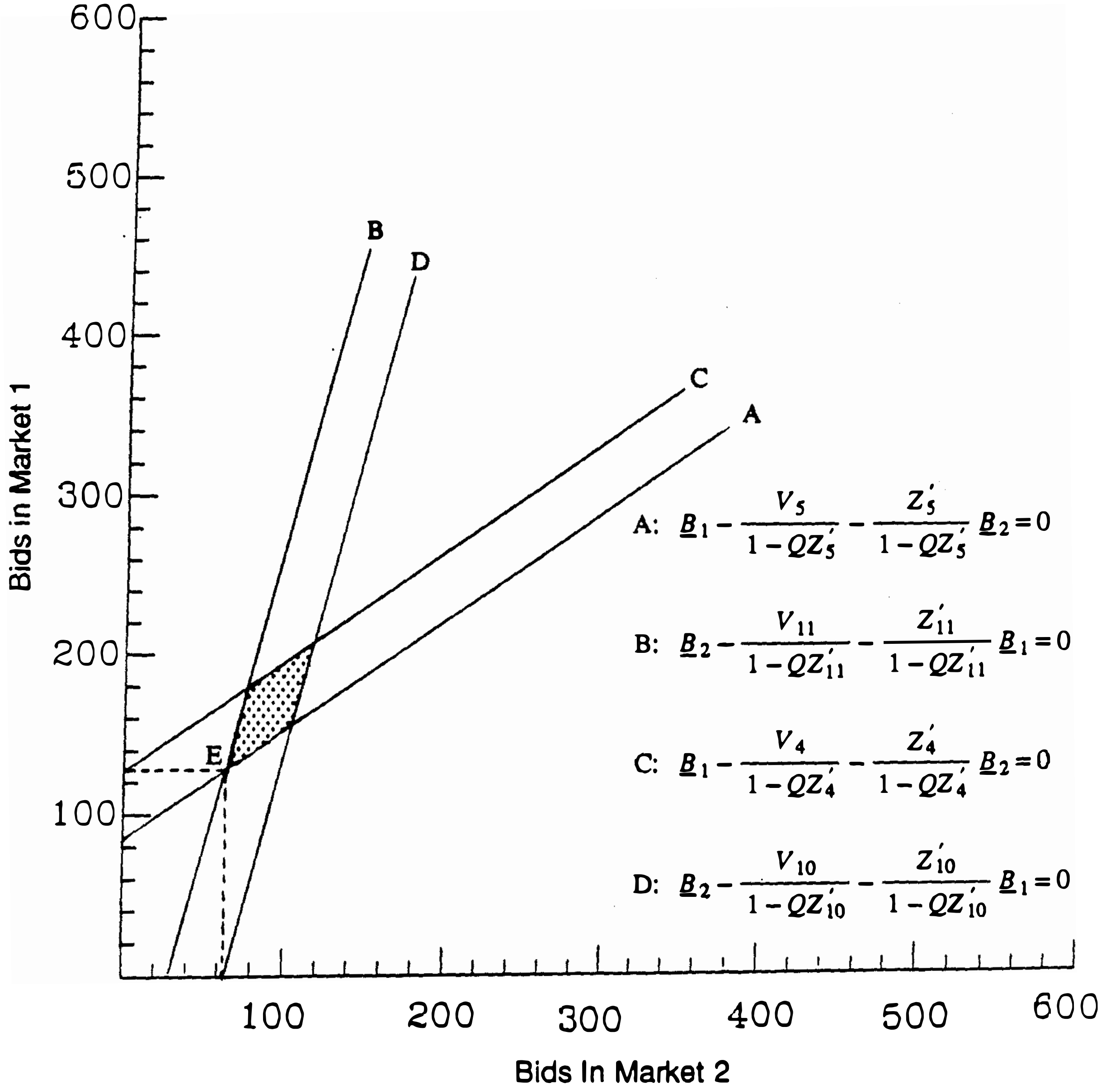


FIGURE 2: Nash Equilibria Parameter Set 2

Thus the static game model predicts winning bids that are inside the rhomboid. The lower left point of the rhomboid is predicted by a special dynamic model and it is the only "trembling hand" Nash equilibrium.

In order to justify the geometry, a formal development of the model is necessary.

A1. The day is divided into $K = 2$ periods.[2] A total of $Q = 4$ slots is available in each period. $K(Q + 2)$ carriers are exogenously partitioned into K submarkets corresponding to the K periods. In each submarket there are $Q + 2$ bidders. There is no aftermarket.

Let

$J = \{1,2, \ldots, 12\}$ = index set of carriers

$K = \{1,2\}$ = index set of submarkets

V_j = value of a slot to firm j

X_j = the enplaned passengers (capacity) of carrier j should it be allocated a slot

B_j = the bid tendered by carrier j

$k(j)$ = the submarket in which carrier j participates. $k(j) = 1$ for $j \in \{1,2,3,4,5,6\}$ and $k(j) = 2$ for $j \in \{7, 8, 9, 10, 11, 12\}$.

$\underline{B}_k$ = maximum rejected (second lowest) bid in submarket k

W_k = the set of carriers that tendered winning bids in submarket k. If there are no ties, then $W_k = \{j \in J : k(j) = k \text{ and } B_j > \underline{B}_k\}$

$W = W_1 \cup W_2$ = auction winners

$R = \sum_{j \in W} B_j$ = revenue

$Z_j = \dfrac{X_j}{\sum_{i \in W} X_i}$ = rebate factor of carrier j should it win a slot

A2. Each firm wants only a single slot in the submarket it is operating. It places no value on additional slots or on slots at any other time of day.

Using the above notation we have

$$\Pi_j = \text{profit of carrier } j = \begin{cases} V_j + Z_j R - B_j & \text{if } j \in W \\ 0 & \textit{otherwise} \end{cases}$$

Note that $\sum_{j \in W} Z_j = 1$; that is, all the revenue from the auction is rebated to the winning carriers. The firms are indexed so that if $i > j$, then $V_i \geq V_j$ for all i, j such that $k(i) = k(j) = k$ for all $k \in K$.

A3. $X_i \geq X_j$ if $i < j$ for all j such that $k(i) = k(j) = k$ for all $k \in K$.

A4. Ties are broken randomly.

Let $B = (B_1, \ldots, B_{12})$ be a vector of bids. Note that the winning set W may be random because of A4. Define $(B_j';B)$ to be the vector, B, in which the jth component is replaced by B_j'.

Definition. B^* is a Nash equilibrium if $E\Pi_j(B^*) \geq E\Pi_j(B_j; B^*)$ for all B_j for all $j \in J$. The notation E denotes an expected value.

That is, we say that B^* is a Nash equilibrium if, given that others use the bids given by B^*, no bidder j can be made better off in an expected value sense by tendering a different bid $B_j \neq B_j^*$. Let W_k^*, $k \in K$, denote the set of winning bidders in a Nash equilibrium in submarket k, and W^* denote the set of winners in both submarkets. No confusion should develop from possible nonuniqueness of Nash equilibria. The notation is relative to a given Nash equilibrium.
Let

$$Z_j^* = \begin{cases} \dfrac{X_j}{\sum_{i \in W^*} X_i} & \text{if } j \in W^* \\ 0 & \textit{otherwise.} \end{cases}$$

Let $\underline{B}_k^*$ and $\bar{B}k^*$ be respectively the maximum rejected bid and the minimum accepted bid in market k at Nash equilibrium, i.e., $\underline{B}_k^* = \max\{B_j^* : j \notin W_k^* \text{ and } k(j) = k\}$ and $\bar{B}_k^* = \min\{B_j^* : j \in W_k^* \text{ and } k(j) = k\}$.

First we make several observations about the Nash equilibria given assumptions A1-A4.

(i) At a Nash equilibrium all winners in a given market bid the same amount, namely $\bar{B}_k^*$. Suppose not. Then the agent bidding higher than the other winners could reduce his bid by some small amount and still be among the winners, thereby increasing his/her profit.

(ii) Nash equilibrium winning bidders in each market are the bidders with highest valuations. This is true because the valuations and capacities are ordered in the same way (A3), so that given (i) above, if one or more of the carriers with valuations among the top four in a market were not in W_k^*, then at least one carrier with lowest valuation, say j, is in W_k^* and either : 1) j is making negative profit, in which case he/she could strictly increase his/her profit by bidding zero, or 2) j is making nonnegative profits, in which case one of the excluded bidders (which has higher V and X than does j) could bid some small amount above B_j which would guarantee him a slot, given the bid of others, and make his profits strictly higher.

(iii) At a Nash equilibrium one has $\underline{B}_k \neq \bar{B}_k^*$. There can be no ties to determine which bidder is in W. To see this use (i) above, and follow the same reasoning as in (ii).

From these observations we conclude that at a Nash equilibrium, if one exists, the winning set W^* is nonrandom. In each submarket k the four bidders with the highest valuations are the winners, i.e., $W_1^* = \{1,2,3,4\}$ and $W_2^* = \{7,8,9,10\}$. Moreover, the winners tender a common bid $\bar{B}_k^*$ strictly greater than the maximum $\underline{B}_k^*$ of the bids of the two buyers with lowest valuations.

Under assumptions A1-A4 existence and uniqueness problems occur immediately due to the discontinuity of the payoff function, i.e., each winning player has incentive to get as close as possible to $\underline{B}^*_k$ but not bid $\underline{B}^*_k$ because then a tie and a lottery will occur. Since each player is attempting to minimize a bid over an open set, no equilibrium exists. There are several ways to avoid this problem: inherent discreteness of the bid space, some randomness due to mixed strategies, "trembling hands," or randomness in the process itself can produce equilibrium. In our case, the first reason is sufficient so that we have

$$\bar{B}^*_k = \underline{B}^*_k(1+\varepsilon) \qquad \text{for } all\ k \in K \tag{1}$$

where ε is an arbitrarily small, positive quantity.

We shall now show that the set of Nash equilibria is the rhomboids of Figure 1 and 2. Let R^* be the Nash equilibrium revenue, i.e.,

$$R^* = Q \sum_{k \in K} \bar{B}^*_k .$$

From the definition of Nash equilibrium it is clear that winners must prefer winning to nonparticipation:

$$V_j + Z^*_j R^* - \bar{B}^*_{k(j)} \geq 0 \quad \text{for } all\ j \in W^*, \tag{2}$$

and for carriers $j \notin W^*$ it should be the case that bidding $\bar{B}^*_{k(j)}$ gives *expected* nonpositive profit, i.e.,

$$V_j + Z'_j Q\,[\sum_{m \neq k(j)} \bar{B}^*_m + (Q-1)\bar{B}^*_{k(j)}] - \bar{B}^*_{k(j)} \leq 0 \tag{3}$$

where Z'_j is the equal weight average of $X_j/\Sigma_{i \in W'}$ over all sets W' which differ from W^* in that j replaces one other carrier in $W^*_{k(j)}$. Putting $j = 4{,}10$ in (2) and $j = 5{,}11$ in (3) we obtain the rhomboids of figures 1 and 2. Conversely, it can be shown using assumption A3 that any points in these rhomboids define a Nash equilibrium pair $(\bar{B}^*_1, \bar{B}^*_2)$ of accepted bids.

As was stated earlier the two lines, which are defined by (3) and which are closest to the origin, identify the combinations of winning bids in the two markets that would produce zero expected profits for the extramarginal subjects. Alternatively, these lines define the inframarginal bids $\bar{B}^*_k$ that are sufficiently high to keep the extramarginal agents out of the market. They intersect at a Nash equilibrium that we will call the *Distinguished Nash* equilibrium. The other Nash equilibria are found in the shaded convex body bounded by (2) when applied to $j \in \{4{,}10\}$ and evaluated at zero. They are the prices low enough to keep the inframarginal subjects in the market with nonnegative net profits.

Reasonable assumptions can be used to reduce the set of equilibria. Notice that in the interior of the solution set the inequality (3) is strict inequality. That is, the excluded carriers [who bid $\underline{B}^*_k = \bar{B}^*_k/(1+\varepsilon)$] are bidding so high that should they replace one of the winning carriers, they would make negative profits on average. If some chance actually existed that these excluded bidders might win, e.g., trembling hand, the maximum they would bid would be given by the equality in (3). This observation reduces the set of equilibria to a single point E^* in the figure that we call the

Distinguished Nash. In this case equilibrium is given by the solution to a set of two equations in two unknowns $(\bar{B}_1^*, \bar{B}_2^*)$. Specifically, from (3) written for $j = 5, 11$, we obtain:

$$\begin{aligned} \bar{B}_1^* &= V_5 + Z_5' Q(\bar{B}_1^* + \bar{B}_2^*) \\ \bar{B}_2^* &= V_{11} + Z_{11}' Q(\bar{B}_1^* + \bar{B}_2^*) \end{aligned} \tag{4}$$

The solution is given by

$$\begin{aligned} \bar{B}_1^* &= V_5 + QZ_5' \frac{V_5 + V_{11}}{1 - Q(Z_5' + Z_{11}')} \\ \bar{B}_2^* &= V_{11} + QZ_{11}' \frac{V_5 + V_{11}}{1 - Q(Z_5' + Z_{11}')} \end{aligned} \tag{5}$$

The solution is *always* strictly positive because $Q(Z_5' + Z_{11}') < 1$ as can readily be shown using the definition of Z_j'. So, at the Distinguished Nash equilibrium, the four bidders with the highest four valuations in market k bid $\bar{B}_k^*$. By definition, at least one of the two bidders with lowest valuations in market k bids $\underline{B}_k^* = \bar{B}_k^*/(1 + \varepsilon)^3$. Since these two bidders are actually identical (see Table 1), by symmetry we shall consider that both extramarginal agents bid $\underline{B}_k^*$ at the distinguished Nash equilibrium.

These Distinguished Nash equilibrium bids $\bar{B}_k^*$ are given in Table 2. The table also gives the demand and supply equilibrium values that would exist in the absence of the rebate or zero-out feature. As can be seen from (5), the Distinguished Nash equilibrium of the zero-out auction gives rise to prices above those which would result if a regular auction were used. That is, the equilibria under the zero-out feature are always larger than (V_5, V_{11}) which would be the competitive equilibrium in the absence of the rebate feature.

SECTION IV. EXPERIMENTAL DATA

The experimental results are given in Figures 3 through 8. Shown there are the time series of bids tendered by all individuals in all markets. The dotted horizontal lines are the equilibrium prices corresponding to the Distinguished Nash equilibrium E in markets 1 and 2 for each experiment taken from Table 2. Leaving the details of the analysis of the data to the next section, we observe:

Conclusion 1. In the first set of experiments (1, 2, 3) we do not see any convergence to a Nash equilibrium. In experiments 4, 5, and 6 bids eventually settle down near the Distinguished Nash equilibrium levels.

Support: The figures clearly make the point. Prices in all markets of experiments two and three exceed the Nash equilibrium by nearly a factor of two. Furthermore, the trend does not seem to be in the direction of equilibrium. The final prices in experiments 4, 5, and 6 all appear to be stabilizing around a value "close" to the Distinguished Nash equilibrium. Detailed statistical analysis will be presented later.

Table 2: Equilibrium Prices

Experiment	Market	Competitive Equilibria Without Rebate	Distinguished Nash Equilibrium
1, 2, 3	Market 1	50	307
	Market 2	25	196
4, 5, 6	Market 1	50	128
	Market 2	25	63

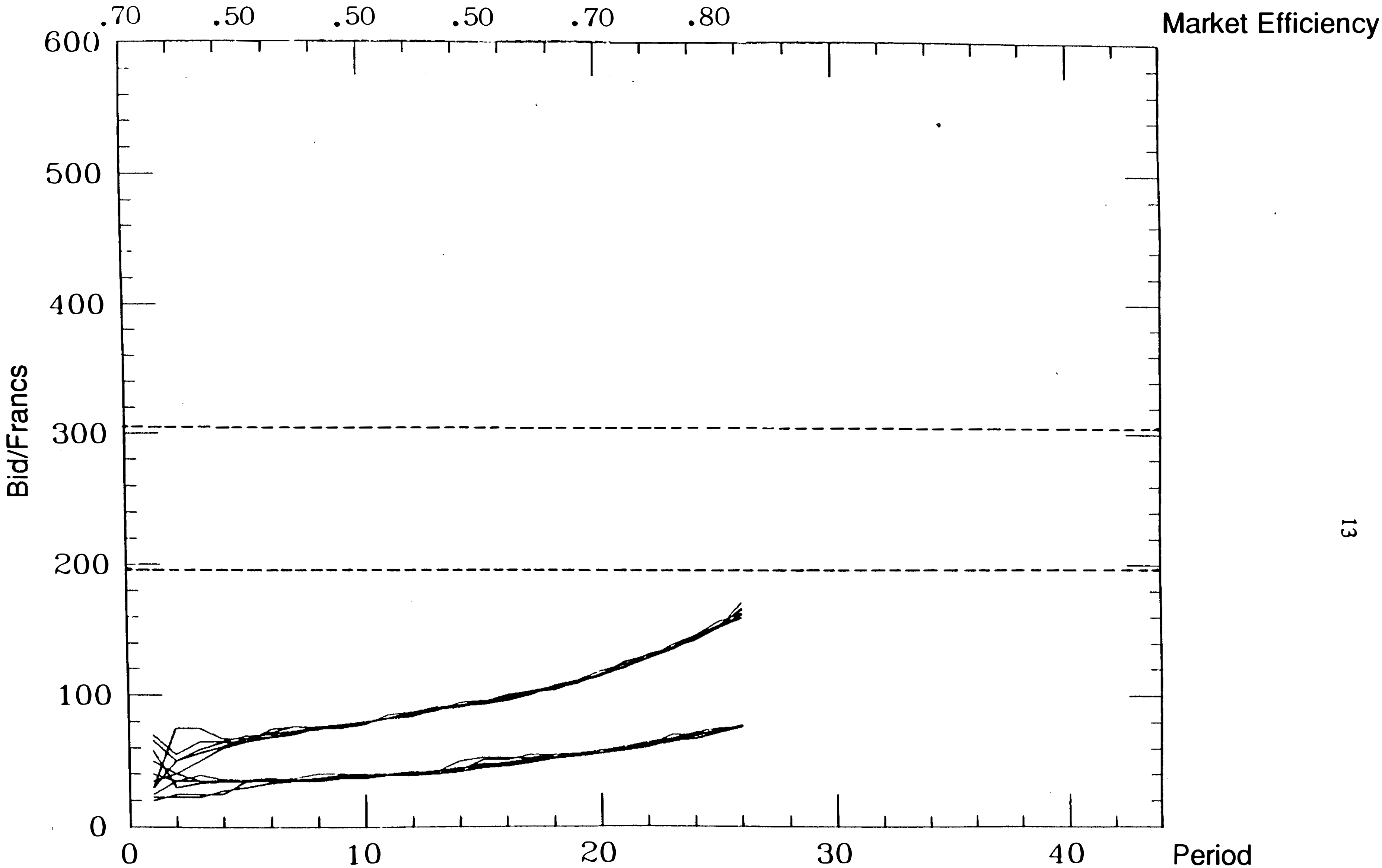


FIGURE 3: Experiment 1. Market Efficiency Levels and Bids by Each Individual, Two Markets, All Periods

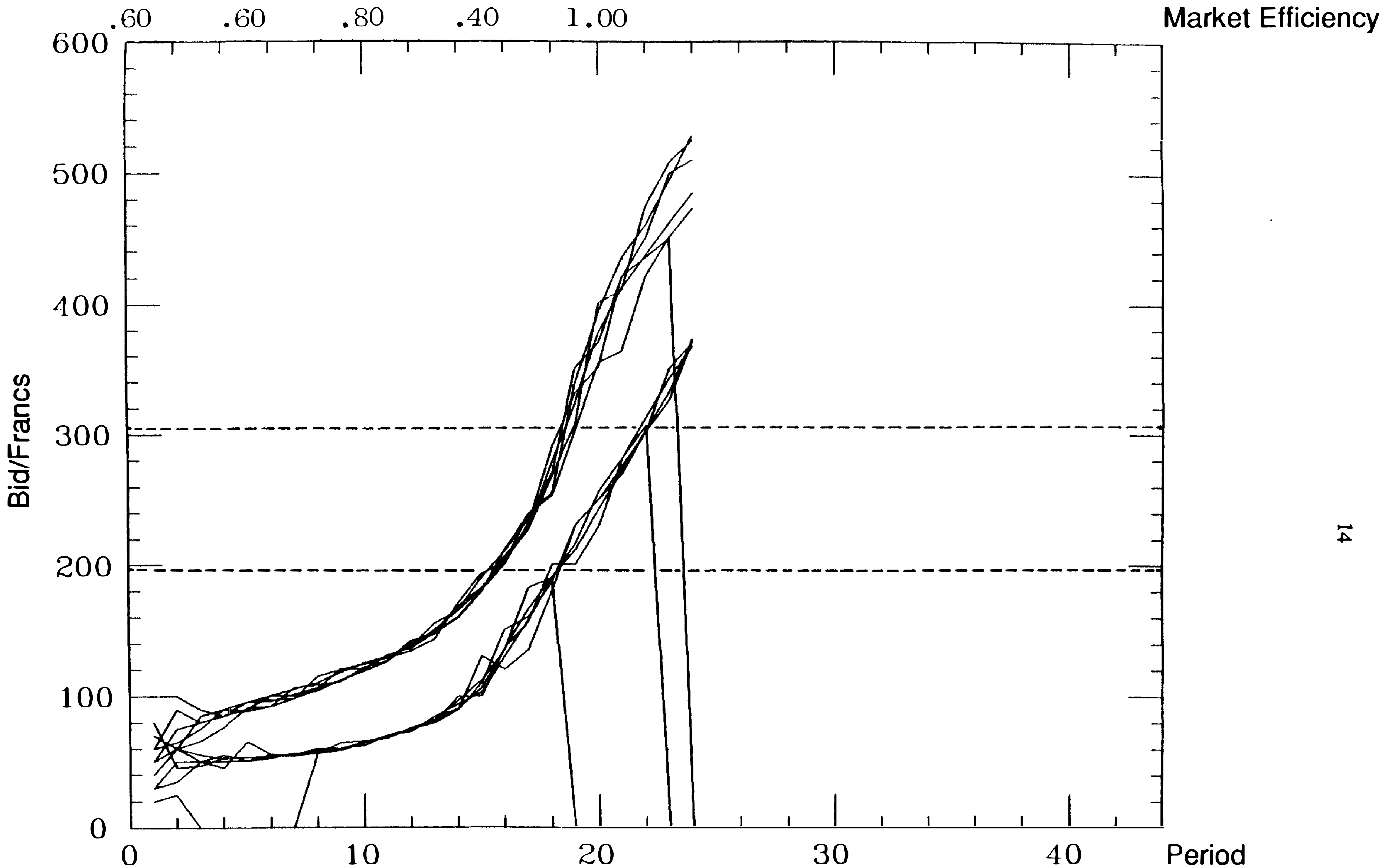


FIGURE 4: Experiment 2. Market Efficiency Levels and Bids by Each Individual, Two Markets, All Periods

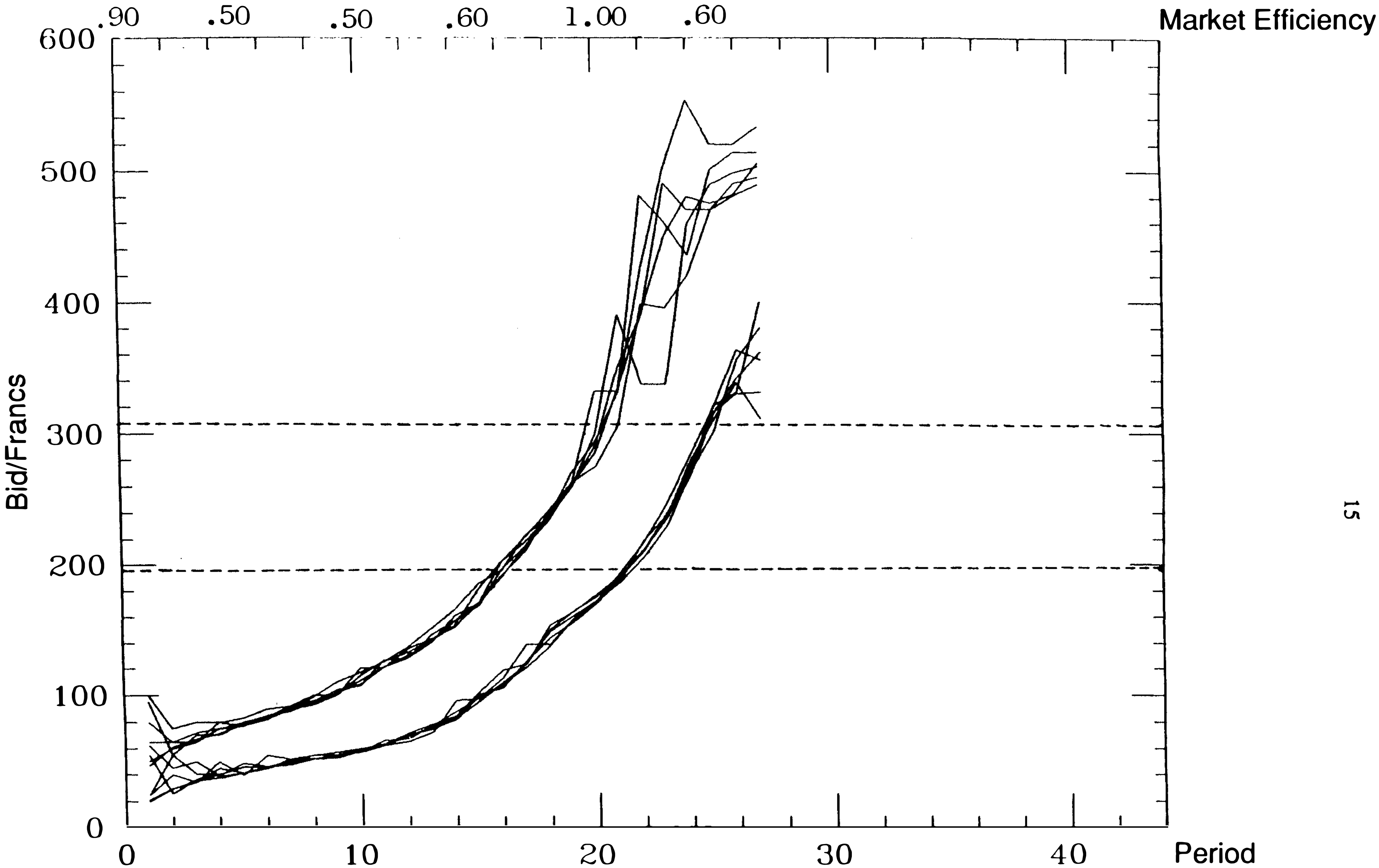


FIGURE 5: Experiment 3. Market Efficiency Levels and Bids by Each Individual, Two Markets, All Periods

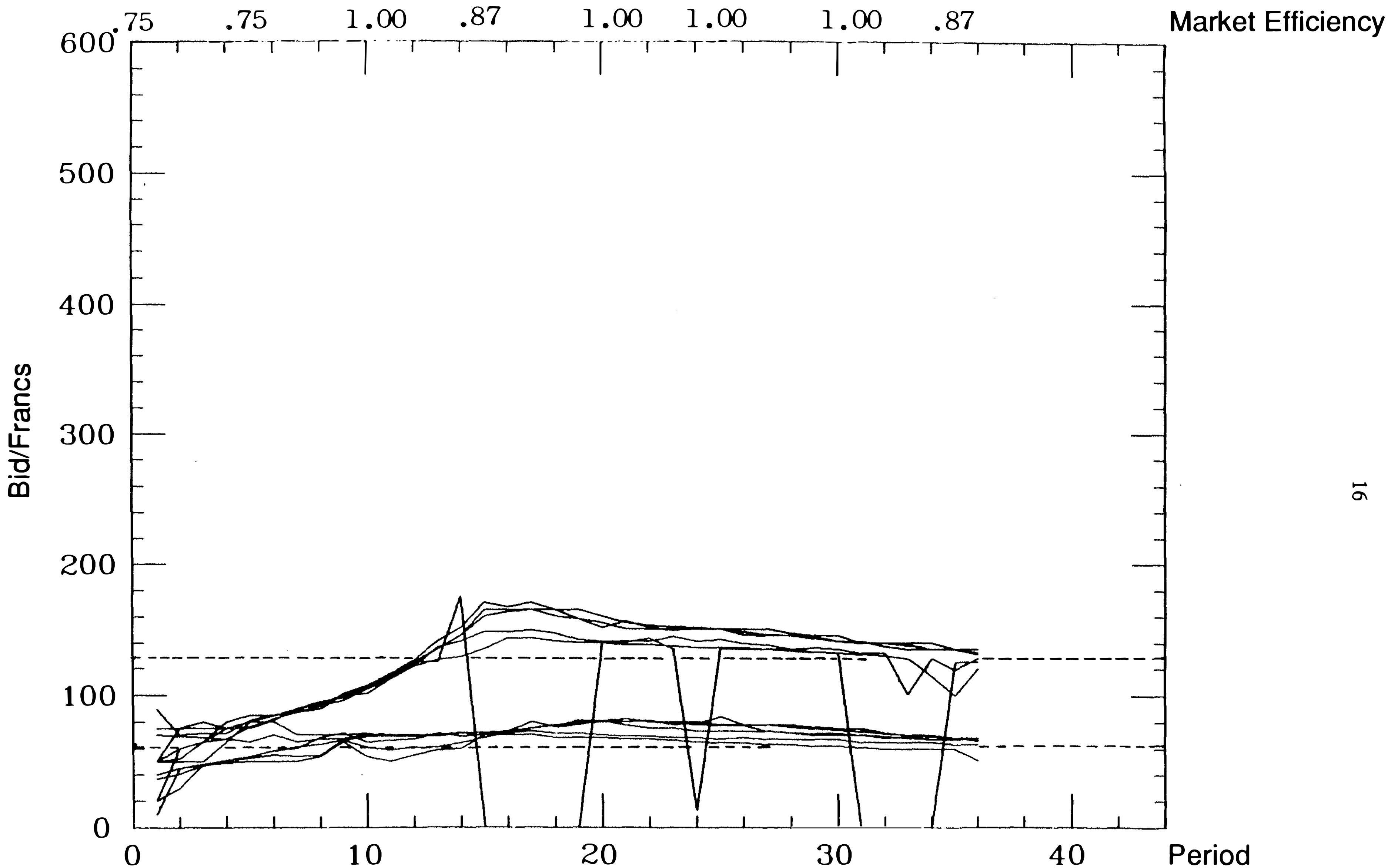


FIGURE 6: Experiment 4. Market Efficiency Levels and Bids by Each Individual, Two Markets, All Periods

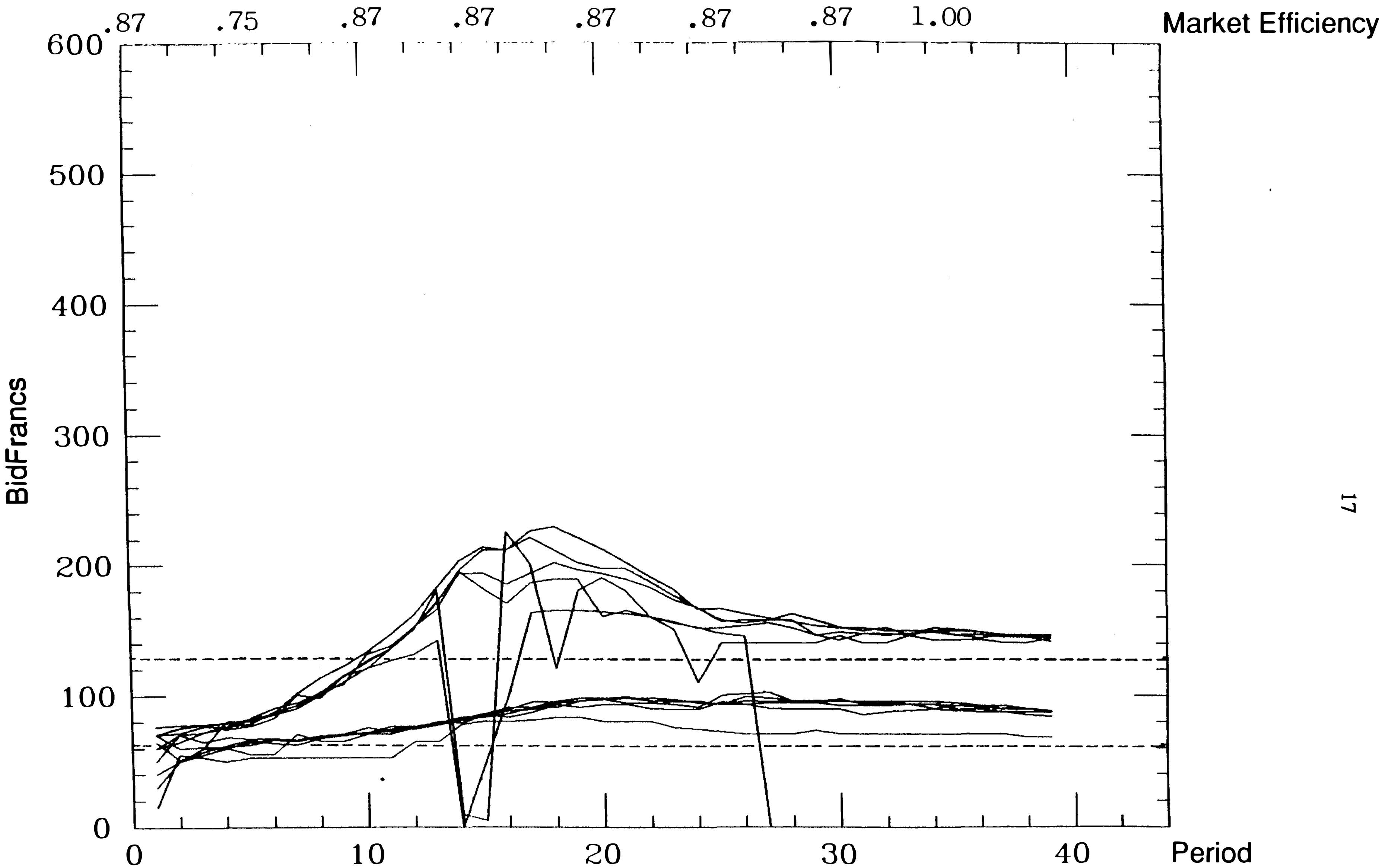


FIGURE 7: Experiment 5. Market Efficiency Levels and Bids by Each Individual, Two Markets, All Periods

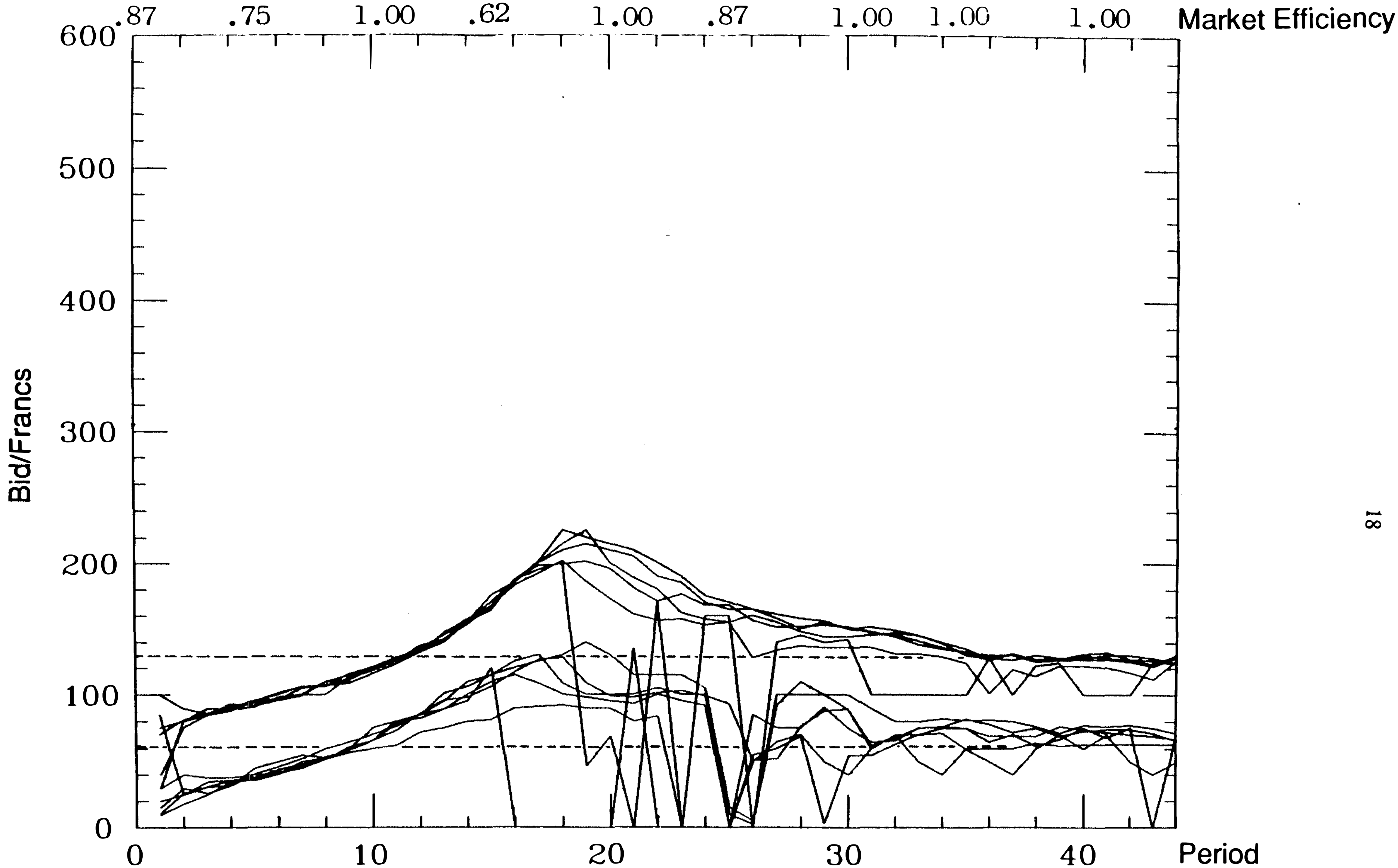


FIGURE 8: Experiment 6. Market Efficiency Levels and Bids by Each Individual, Two Markets, All Periods

Efficiency levels for selected periods are presented at the top of Figures 3 through 6. As an effi measure we use the ratio of "the difference between the total valuation of the winners and lowest possible total valuation" to the "difference between highest and lowest possible total valuations." The average efficiency ratio is in the vicinity of 64 percent for the first set of experiments and over 85 percent for the second set. In general, Market 2 seems to be more efficient than Market 1 but the difference is insignificant and the separate numbers are not presented. As the process approaches the Distinguished Nash equilibrium as in experiments 4, 5, 6, the efficiency increases to near 100 percent.

The data dramatically demonstrate support for one prediction of the Nash equilibrium model.

Conclusion 2. Inflation will occur over what would have been the prices without the rebate.

Support: Compare the Nash equilibrium range to the competitive equilibrium prices in Table 2. The data are near the Nash equilibria so the data demonstrate that the Nash model does have some merits.

A glance at the data reveal that the process is characterized by dynamics that are beyond the scope of a static Nash equilibrium model. From the first three experiments an "explosive" possibility is evident. The purpose of the following section is to explore the nature of the dynamics of the decisions which generated these time series.

The data from experiments 1, 2, 3 were available when parameter set 2 was derived. The motivation for the second parameter set involved distinguishing among hypotheses that were developed from the experimental data obtained from parameter set one. The discussion of data analysis below reflects this sequential decision process regarding the experimental design.

SECTION V. ADJUSTMENT PROCESSES

Clearly the data do not adjust immediately to the predictions of a static game model. Some sort of dynamic process is operating. Convergence to the Nash equilibrium is a complex phenomenon. Only a few theoretical papers have proposed some dynamic rational models, which are quite complicated even for the simplest of markets (Easley and Ledyard 1986, Friedman 1984, Wilson 1985, forthcoming (a) and forthcoming (b)). In this section, we propose and test against the data four simple dynamic processes to explain the behavior of bids over time. Model 1 is unconstrained by game-theoretic principles. Each of Models 2, 3, and 4 retain only some aspects of the one-shot Nash equilibrium and therefore are not fully rational. Though the proposed models are relatively simple, our position is that their validity relies on relative, satisfactory, explanatory, and predictive power of the observed dynamics. In addition, existence of simple decision rules, which over time give rise to the Nash equilibrium outcomes, would be important in explaining the previous experience of convergence to the Nash equilibrium in experimental markets and in weakening the rationality and informational requirements for the application of the Nash equilibrium concept.

For our purposes static game-theoretic principles fall into two categories: (i) conditions for equilibrium or system-wide consistency, and (ii) conditions for individual optimization. The former category serves as the basis for the traditional approach to modeling dynamic adjustment processes that we employ in subsection A. The latter has the foundations for the approach we propose in subsection B. In both cases, however, the dynamics are introduced through expectations.

A. *General Linear Adaptive Processes*

1. Models 1 and 2:

In these models we assume that each bidder $i \in J$ follows a bidding rule which is linear in his/her expectation of the revenue from the auction at time t. As we shall see below, a justification for such a linear decision rule is consistent with the static Nash equilibrium model. With respect to expectations, we assume that their formation is adaptive. Adaptive expectations first introduced by Cagan (1956) and Nerlove (1956), have been found in applied econometric studies to be quite successful and to outperform other types of expectations. Whether adaptive expectations or rational expectations will prove best is a topic of current research (see Williams 1987 and Daniels and Plott 1987). Formally we assume:

B1. Linear decision rules:

$$B_{it} = \alpha_i + \beta_i R_{it}^* \qquad t = 1, 2, \ldots; \ \beta_i > 0. \tag{6}$$

Or, in terms that can be estimated

$$B_{it} = \alpha_i + \beta_i R_{it}^* + u_{it} \qquad t = 1, \ldots, T; \ i = 1,2, \ldots, 12. \tag{6'}$$

where u_{it} is an error term.

C1. Adaptive expectations formation:

$$R_{it}^* = (1 - \lambda_i)R_{t-1} + \lambda_i R_{i,t-1}^*. \tag{7}$$

The two properties together generate the first model.

Model 1: The *general linear adaptive model* consists of B1 and C1.

The model as stated simply postulates mechanical decision rules that do not depend on game-theoretic principles. The assumption B1 says that bids are derived as a linear markup of the expected rebate. Assumption C1 says that each bidder revises his/her expectation at time t by a function of the discrepancy between the actual revenue and the expected revenue in the previous period. C1 gives a sequence of expected revenue for each $i \in I$ defined by

$$R_{it}^* = (1 - \lambda_i)\sum_{s=0}^{t-2} \lambda_i^s R_{t-1-s} + \lambda_i^{t-1} R_{i1}^*. \tag{8}$$

C1 and B1 together give a sequence of auction revenues over time which is described by the function

$$R_t = \sum_{i \in W_t} \alpha_i + \sum_{i \in W_t} [\beta_i (1 - \lambda_i) \sum_{s=0}^{t-2} \lambda_i^s R_{t-1-s}] + \sum_{i \in W_t} \beta_i \lambda_i^{t-1} R_{i1}^*. \tag{9}$$

where W_t is the set of winners at time t.

A reasonable additional restriction to add to C1 is $0 \leq \lambda \leq 1$. This restriction is not added at the axiomatic level but it is discussed later with the estimation results. The following restriction is useful.

C2. Common expectation adjustments:

$\lambda_i = \lambda$ for all $i \in J$.

The second model to be developed now rests in part on game-theoretic principals. Two behavioral principles that are taken from the properties of static Nash equilibrium behavior are added to the general linear adaptive model to form a second model. Let $\underline{V}_{k(i)}$ denote the value of a slot of the marginal agent in the submarket to which agent i belongs and let $\underline{Z}'_{k(i)}$ be the rebate factor of the marginal agent as defined in (3). The two new properties are:

B2. In each market the marginal bidders bid their value plus their expected rebate; i.e., $\alpha_i = \underline{V}_{k(i)}$ and $\beta_i = \underline{Z}'_{k(i)}$ for the excluded bidders in each market for all t.

Assumption B2 requires that the marginal bidders, those that are excluded from the set of winners in the Nash equilibrium, bid the maximum possible subject to a no expected loss constraint. The excluded bidders bid as high as they can in their effort to become winners so α is at the level of the value of a slot, V_i; and β_i is a correctly calculated rebate factor.

B3. Infra-marginal bidders bid "just above" what they expect to be the marginal's bid, i.e., in (6)

$$\alpha_j = (1+\delta)\underline{V}_{k(j)} \quad \text{and} \quad \beta_j = (1+\delta)\underline{Z}'_{k(j)} \quad \text{for } j \in W$$

for all t, where δ is "small."

Assumption B3 together with B1, B2, C1, C2, imply that the winners bid the minimum necessary to remain winners and place their bids only δ above the bid of the losers. If the dynamic process described in Model 2 defined below converges to the Distinguished Nash equilibrium, then δ is equal to ε defined earlier.

Model 2. The *strategic linear adaptive model* consists of C1 and B1-B3.

The idea of Cournot expectations hypothesis has a long history in the development of economic theory. In the context of the linear adaptive models, the hypothesis has a very simple representation.

C3. Cournot expectation hypothesis.

$\lambda_i = 0$ for *all* $i \in J$.

The hypothesis maintains that each decision maker assumes that the decisions of others will remain the same as the previous period. Obviously C3 implies C2.

Adding C3 to the assumptions adds a substantial restriction to the strategic linear adaptive model. In this case we obtain a closed form expression for the sequence of bids under B1-B3. In this case $R_{it}^* = R_{t-1}$ for all bidders and the following proposition can be derived.

Proposition 1. Under C1-C3 and B1-B3 the sequence of bids for the *marginal bidders* (agents 5, 6, 11, 12) satisfy

$$\underline{B}_k(t) = \underline{V}_k + \underline{Z}_k'\{R(0)[Q(1+\delta)(\underline{Z}_1' + \underline{Z}_2')]^{t-1} + [(\underline{V}_1 + \underline{V}_2)/(\underline{Z}_1' + \underline{Z}_2')]\sum_{s=1}^{t-1}[Q(1+\delta)(\underline{Z}_1' + \underline{Z}_2')]^s\} \tag{10}$$

where $R(0)$ = auction revenue in initial period.

From equation (10) we have

$$\begin{aligned}\Delta\underline{B}_k(t) &= \underline{B}_k(t) - \underline{B}_k(t-1)\\ &= [Q(1+\delta)(\underline{Z}_1' + \underline{Z}_2')]^{t-1}\{\underline{Z}_k'R(0)[Q(1+\delta)(\underline{Z}_1' + \underline{Z}_2') - 1] + Q(1+\delta)\underline{Z}_k'(\underline{V}_1 + \underline{V}_2)\}.\end{aligned} \tag{11}$$

Proposition 2. $\underline{B}_k(t)$ converges to an assymptote if and only if

$$0 \leq |Q(1+\delta)(\underline{Z}_1' + \underline{Z}_2')| < 1 \tag{12}$$

Taking logarithms of both sides and adding an error term we get a form that can be estimated:

$$\log \Delta\underline{B}_k(t) = \alpha + \beta t^* + u(t),\ u_t \sim iid\ N(0,\sigma^2) \tag{13}$$

where

$$t^* = t - 1$$

$$\alpha = \log\{\underline{Z}_k R(0)[Q(1+\delta)\underline{Z}_1' + \underline{Z}_2') - 1] + Q(1+\delta)\underline{Z}_k'(\underline{V}_1 + \underline{V}_2)\}$$

$$\beta = \log[Q(1+\delta)(\underline{Z}_1' + \underline{Z}_2')].$$

The condition for the convergence of $\underline{B}_k(t)$ given by (12) and referenced in Proposition 2 translates into the condition that β, the coefficient of t^* in (13), is negative. This forms the basis of tests to be performed below.

2. Data Analysis: Experiments 1, 2, and 3.

The data for experiments 1, 2, and 3 are analyzed separately. This treatment permits a deeper analysis of the models, the estimation techniques and the motivation for parameter set 2, which was used in experiments 4, 5, and 6.

As seen in Figures 3 through 5 we do not observe convergence to the Nash equilibrium in the fi three experiments contrary to previous experience in experimental auctions. In experiment 1 the bids, although increasing, remain well below the Nash equilibrium levels in both markets during the entire session. In experiments 2 and 3 we observe a faster increase in bids and bids remains above the Nash equilibrium level at the end of both experiments. These properties of the data can be expressed more formally by estimations of exponential functions but space demands that the visual impression must suffice and we move immediately to tests of the models.

Equation (13) is a natural first test. It involves the most restrictive set of hypotheses. So the analysis starts with it and then explores relaxations of the assumptions.

Conclusion 3. If the Cournot expectations hypothesis C3 is maintained with the strategic linear adaptive model (Model 2), then the bids do not converge. The variable δ is too large.

Support: This conclusion results from equation (13) estimated separately for each submarket using the logarithm of the changes in minimum accepted bid as the dependent variables.[4] The regression results are given in Table 3. In all cases the coefficient of t^* is significantly greater than zero which, maintaining assumptions C1-C3 and B1-B3 and using Proposition 2 implies δ is, on the average, too big for convergence of bids. According to this model equilibrium does not exist in the sense of an asymptote of the dynamic process.

Whether or not Conclusion 3 is accepted as an explanation of the first three experiments rests upon the hypotheses on which equation (13) is based. That is, if one or more of those hypotheses (C1-C3, B1-B3) turns out to be violated in the analysis of individual bids, the results based on (13) and reported in Conclusion 3 would not be directly applicable. We now explore the specifi of the most general model in order to determine which "parts" of (13) are not reliable.

The general linear adaptive model consists of only B1 and C1, which in (6′) was written as:

$$B_{it} = \alpha_i + \beta_i R_{it}^* + u_{it} \quad t = 1, \ldots, T; \quad i = 1, 2, \ldots, 12 \tag{14}$$

We assume that for every i, $u_{it} \sim iid\ N(0, \sigma_i^2)$.[5] The parameters to be estimated are

$$(\alpha_i, \beta_i, \lambda_i, \mu_i, \sigma_i) \qquad i = 1, 2, \ldots, 12$$

where $\mu_i = y_{i1} - u_{i1}$. See Appendix A for the estimation procedure.

Maximum likelihood estimates are presented in Table 4a and 4b. The R^2s are very high. For fi of the seventy-two individuals the R^2s are over .90 and for fifty-two of the seventy-two the R^2s are over .95. As a further check on the accuracy of the model we perform two specification tests on the general validity of (14) and of the statistical assumptions on the error terms.

Table 3: *OLS* Estimates

Exp. No.	Market	$\hat{\alpha}$	$\hat{\beta}$	R^2	$t_{\hat{\beta}}$
1	1	1.1751 (.2220)	.0229 (.0149)	.0932	1.538
1	2	.2142 (.2203)	.0338 (.0143)	.2105	2.366
2	1	1.5671 (.3232)	.0903 (.0163)	.5942	5.545
2	2	.7239 (.3434)	.1264 (.0240)	.5934	5.266
3	1	1.5585 (.2437)	.0731 (.0158)	.4720	4.632
3	2	.7999 (.1758)	.1026 (.0113)	.7721	9.018

$\log(B_t - B_{t-1}) = \alpha + \beta(t-1) + u_t$

B_t = minimum accepted bid at time t

Table 4a: General Linear Adaptive Model—Experiments 1, 2, 3
(Standard deviations are below the coefficient estimates)

Exp. No.	Subject	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\mu}$	R^2	H*	G**
1***	1	−5.4190	.1783	−.9962	74.8520	.9983	.0161	1.6835
		.5527	.0010	.0069				
1	2	−12.5300	.2032	.5703	82.1299	.9965	.1166	.0678
		3.0773	.0079	.0858				
1	3	−11.7263	.1980	.4882	32.1495	.9975	.0002	6.2401
		.8369	.0014	.0144				
1	4	−12.5531	.1994	.4727	9.9339	.9960	.0585	.6264
		.8768	.0014	.0128				
1	5	−5.6230	.1863	.3164	11.0098	.9968	.0046	1.8686
		.8129	.0013	.0075				
1	6	−6.2064	.1887	.4688	5.9680	.9974	.0359	2.9987
		.8932	.0015	.0180				
1	7	−5.0291	.0975	.5664	44.3921	.9982	.1909	4.7370
		1.4013	.0029	.0425				
1	8	1.9327	.0839	−.0038	1344.1782	.9909	.0011	4.9076
		.7485	.0013	.0002				
1	9	−5.7821	.0970	.5977	14.7919	.9971	.0283	.0812
		.9342	.0020	.0567				
1	10	−9.1632	.1083	.7500	36.0427	.9969	.0604	.0526
		7.5172	.0195	.1475				
1	11	−4.3978	.0972	.6172	16.9288	.9901	.0061	.0095
		.9361	.0019	.0523				
1	12	−6.2537	.1051	.7266	37.3638	.9900	.0017	.2304
		55.7683	.1474	1.3781				
2	1	3.5287	.1610	−.2188	143.5540	.9955	.0063	1.4063
		2.7627	.0023	.1117				
2	2	5.1388	.1590	−.4063	121.9938	.9952	.0010	1.3536
		2.0355	.0017	.1171				
2	3	10.5069	.1526	−.6953	82.1366	.9943	.0280	.0149
		2.1064	.0009	.0259				
2	4	15.8830	.1441	−.8868	77.2287	.9946	.0167	14.0848
		.9561	.0041	.9118				
2***	5	16.1861	.1363	−.9962	79.0411	.9912	.0389	7.1974
		3.4217	.0018	.0874				
2	6	67.8549	.0899	−.7382	111.3581	.4926	.2561	5.6743
		2.5067	.0015	.2134				

* H = Hausman statistic
** G = Godfrey statistic
*** Maximum likelihood does not converge.

Table 4a: General Linear Adaptive Model—Experiments 1, 2, 3 (*continued*)
(Standard deviations are below the coefficient estimates)

Exp. No.	Subject	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\mu}$	R^2	H*	G**
2	7	−14.0196	.1121	−.8555	34.0191	.9985	.0052	2.8709
		.3904	.0004	.0813				
2	8	−10.0564	.1083	−.9532	35.5939	.9955	.0514	5.0868
		1.2473	.0005	.0222				
2***	9	−9.8516	.1073	−.9962	39.3101	.9931	.0791	.0021
		.2939	.0004	.0899				
2	10	−14.4853	.1130	−.7539	40.4785	.9945	.0000	1.2926
		.9884	.0005	.0238				
2***	11	24.6861	.0498	−.9962	42.3778	.2829	.0001	9.9462
		4.9773	.0020	.0572				
2***	12	4014.2335	−3.5545	.9962	20.7822	.4736	−.0158	4.5497
		31051.7027	27.9806	.0300				
3	1	13.2812	.1558	−.7734	120.2568	.9715	.0017	.6698
		1.7821	.0007	.0293				
3	2	6.6348	.1648	−.7070	118.8238	.9849	.0010	8.7551
		.4575	.0002	.0139				
3	3	19.0295	.1510	.1407	−185.6034	.9753	.0553	1.1767
		1.1578	.0006	.0109				
3	4	10.9858	.1518	−.8594	105.7370	.9848	.0156	13.2137
		2.0206	.0008	.0155				
3	5	16.8180	.1423	−.6719	131.3930	.9895	.0005	.6396
		1.8375	.0009	.0739				
3	6	12.4340	.1502	−.7539	114.5557	.9903	.0144	12.1468
		2.2525	.0008	.0447				
3***	7	−892.6652	3.3467	.9962	34.3558	.9962	−.2657	1.4643
		2362.6910	8.6388	.0101				
3	8	−7.7792	.1239	.6680	13.7623	.9962	.1756	7.3876
		.5145	.0022	.0212				
3	9	−.8126	.0994	−.0038	−713.6254	.9952	.0005	4.2764
		.3692	.0002	.0004				
3	10	−9.3693	.1152	.4766	40.2377	.9964	.0801	.1289
		.5198	.0009	.0242				
3	11	−4.5426	.0995	−.0038	4863.9260	.9943	.0315	2.3026
		.4412	.0002	.0001				
3***	12	−846.3691	3.3889	.9962	29.7508	.9968	−.0893	3.0076
		2831.1548	11.1072	.0129				

* H = Hausman statistic
** G = Godfrey statistic
*** Maximum likelihood does not converge.

Table 4b: General Linear Adaptive Model—Experiments 4, 5, 6
(Standard deviations are below the coefficient estimates)

Exp. No.	Subject	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\mu}$	R^2	H*	G**
4	1	−25.0547	.1945	−.6132	57.4111	.9864	.0473	15.5491
		3.7849	.0044	.3405				
4	2	−43.0220	.2182	−.7031	53.5975	.9826	.0177	14.8688
		.8129	.0010	.0282				
4	3	−32.5642	.2051	−.3555	81.0979	.9894	.0012	7.0440
		2.5932	.0031	.0877				
4***	4	−13.5749	.1700	−.9962	60.4507	.9670	.0092	.8505
		.4151	.0006	.0160				
4	5	123.8565	−.0501	.5586	8.8309	.0744	.0023	7.9155
		16.4980	.0203	.1181				
4	6	−1.6804	.1526	−.3868	55.1355	.9685	.0001	6.5461
		1.6689	.0019	.1035				
4	7	60.3290	.0162	.4336	19.9793	.9044	.0400	10.9132
		1.8302	.0023	.0905				
4	8	34.8851	.0433	.3711	23.2278	.9628	.2001	22.3309
		1.3134	.0015	.0631				
4	9	36.4285	.0436	.5547	26.0304	.9617	.0000	13.8155
		1.5934	.0019	.0248				
4	10	49.7471	.0273	.3828	81.9672	.9383	.0000	18.7834
		1.0692	.0013	.0294				
4	11	29.0271	.0436	−.0038	1141.8633	.9761	.0089	2.1779
		1.6768	.0020	.0007				
4	12	24.2160	.0448	−.1641	137.8976	.9402	.0001	2.8410
		.9185	.0011	.0056				
5	1	−18.7409	.1775	−.7461	66.7613	.9676	.2513	19.7823
		.8668	.0009	.0154				
5	2	−30.1579	.1981	−.7968	64.6306	.9375	.8246	29.5259
		1.5779	.0016	.0203				
5	3	−32.1223	.1959	−.7695	64.2337	.9626	.3865	26.3154
		1.0548	.0011	.0167				
5	4	−3.8840	.1554	−.8828	72.2147	.9433	.0574	18.1912
		1.9468	.0019	.0084				
5	5	43.0128	.0983	.6680	56.0277	.4312	.0114	.4469
		1.0723	.0011	.0100				
5	6	18.7222	.0616	−.7500	48.6479	.0685	.0229	26.4481
		.7752	.0008	.0156				

* H = Hausman statistic
** G = Godfrey statistic
*** Maximum likelihood does not converge.

Table 4b: General Linear Adaptive Model—Experiments 4, 5, 6 (*continued*)
(Standard deviations are below the coefficient estimates)

Exp. No.	Subject	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\mu}$	R^2	H*	G**
5	7	40.4263	.0489	.7773	49.5196	.9905	.0052	3.7414
		1.1011	.0011	.0063				
5	8	35.1978	.0530	.8086	68.6148	.9774	.0225	8.0284
		1.0842	.0011	.0059				
5	9	45.3153	.0475	.7930	48.9192	.9905	.0009	7.3530
		1.3449	.0013	.0070				
5	10	38.1411	.0512	.7891	57.8280	.9962	.0399	1.9999
		1.0116	.0010	.0069				
5	11	43.1346	.0428	.7657	44.9572	.9802	.0336	.0000
		1.1831	.0012	.0086				
5	12	20.5706	.0507	−.0038	−2316.3515	.9691	.0025	6.4342
		1.8322	.0018	.0004				
6	1	16.2107	.1437	.1914	70.8910	.9647	.0033	11.8734
		1.6112	.0016	.0125				
6	2	9.0558	.1551	.2188	122.2901	.9667	.0000	12.4561
		.8802	.0009	.0123				
6	3	21.5418	.1364	−.0038	1569.8505	.9425	.0011	10.2693
		.6982	.0007	.0002				
6	4	30.0167	.1273	−.0038	2408.2264	.9679	.0003	10.1055
		.6784	.0007	.0002				
6***	5	64.2706	.0550	−.9962	81.9712	.2254	.0017	.6903
		35.6215	.0351	.1003				
6	6	31.7151	.1142	−.3125	119.1288	.9244	.0046	15.8567
		.5789	.0006	.0146				
6	7	−21.8439	.1144	−.6680	36.1015	.7550	.0168	8.7875
		.6952	.0007	.0118				
6	8	−3.7634	.0931	−.7618	51.1217	.6695	.0353	12.7436
		2.1614	.0022	.0210				
6	9	−7.3156	.0950	−.3086	69.2866	.8762	.0234	17.3237
		1.5705	.0016	.0510				
6	10	5.6431	.0745	−.7813	50.0648	.5518	.0035	.8652
		2.3383	.0023	.0160				
6***	11	15.8739	.0451	−.9962	34.8026	.3208	.1076	17.2880
		9.7840	.0094	.0780				
6	12	149.1241	−.1064	.7891	−10.5374	.2765	.1070	.5779
		11.1081	.0118	.0292				

* H = Hausman statistic
** G = Godfrey statistic
*** Maximum likelihood does not converge.

Conclusion 4. Specification tests when applied to experiments 1, 2, and 3 fail to reject (14) and thus lend support for the general linear adaptive model (Model 1).

Support: A Hausman (1978) test and a Godfrey (1978) test were used as specification tests. These results are as follows:

a. Hausman Tests:

For each individual we test whether or not the individual's bid deviates from the model prediction $\alpha_i + \beta_i R_{it}^*$ by an error which is uncorrelated with the individual's expectation of R_t. To perform a Hausman test, we obtain Instrumental Variables estimates of the parameters in addition to the ML estimates. A Hausman test statistic for testing the null hypothesis that the expectation R_{it}^* is uncorrelated with u_{it} is

$$\frac{(\beta_{IV} - \beta_{MLE})^2}{var(\beta_{IV}) - var(\beta_{MLE})} \xrightarrow{D} \chi^2_{(1)} \; under\, H_0.$$

(See Appendix A for details.)

b. Godfrey Test:

For each buyer we test the hypothesis that the best linear predictor of the buyer's bid is $\alpha_i + \beta_i R_{it}^*$ by testing whether the errors u_{it} are white noise. We use Godfrey's statistic since both Durbin-Watson (1950) and Durbin's (1970) h-statistics are inappropriate in that R_{it}^*, by construction, contains lagged endogenous variables. We test the null hypothesis that the errors are white noise against the alternatives

$$H_A^{AR} : u_{it} \sim AR(1)$$

and

$$H_A^{MA} : u_{it} \sim MA(1)$$

The test statistic is $T \times R_i^2$ where R_i^2 is the R^2 obtained from the regression of $\hat{u}_i$ on the RHS of (15) and $\hat{u}_{il}$ where $\hat{u}_{i1} = (0, \hat{u}_{i2}, \ldots, \hat{u}_{iT})$. (See Appendix A for details).

Both Hausman and Godfrey statistics are distributed χ^2 with 1 degree of freedom under the corresponding null hypothesis. Critical values are 2.706, 3.841, and 6.635 for 10, 5, and 1 percent significance levels, respectively. The last two columns in Table 4a contain the values for these statistics. In all cases we fail to reject the null hypothesis of the Hausman test.[6] For the Godfrey test only in seven out of thirty-six cases we reject the null hypothesis at the 1 percent significance level. At the 10 percent significance level, about half the cases survive the Godfrey test. Thus, these two specification tests give support for the general linear adaptive model as claimed in Conclusion 4.

The Cournot expectations assumption which was part of (10), the model that led to Conclusion 3, can now be examined more carefully.

Conclusion 5. The Cournot assumption can be rejected in the presence of the general linear adaptive model.

Support: In Table 5a, for all except two of thirty-six cases, the λ estimates are significantly different from zero.[7]

Observation. A significant proportion of λs are negative in experiments 2 and 3. Since negative λs lead to an interpretation problem, these statistics might form a basis for rejecting the general linear model (at least the standard interpretation of C1) in those two experiments. Experiment 1 does not have this problem.

The picture that emerges so far is clouded. The Cournot expectations model can be rejected, but the jury is out on the rest of the model. The general model has mixed support. On the one hand the specification tests support the general linear model but on the other hand the observation above poses an interpretation problem for the model.

We now turn to a different approach to testing that exists by virtue of the experimental methods used to produce the data. Because the experimental parameters are known, a direct test of the strategic linear adaptive model is possible. No need exists to rely upon specification tests alone. When coefficients of the model are tested against known parameters of the experiment as is indicated by B2 and B3, a different picture emerges The next conclusion suggests that the lack of existence of an asymptote of the dynamic process, which Conclusion 3 offers as an explanation of the first three experiments, cannot be accepted as an explanation.

Conclusion 6. Model 2, the strategic linear adaptive model [(14), B2, B3] can be rejected.

Support: Test statistics for testing B2 and B3 are given in Table 5a. The statistics $t(\hat{\alpha})$ is for testing the null hypothesis that α_i = marginal bidder's value. $t(\hat{\beta})$ is the t-statistic for the null hypothesis that β_i = marginal bidder's rebate factor $(\underline{Z}'_i)$. F statistic is for the null hypothesis that $\alpha_i = \underline{V}_i$ and $\beta_i = \underline{Z}'_i$. In only eight out of thirty-six cases we fail to reject this joint null hypothesis. However, the fact that in five of these eight the maximum likelihood procedure does not converge reduces the number of cases where the joint null hypothesis cannot be rejected to three of the thirty-six cases.

Thus the hypothesis tests lead to a rejection of the strategic linear adaptive model as applied to the first three experiments. The specification test results together with the very high R^2s suggest that B1 alone is not entirely off the mark, but the rest of the adjustment process in the previous section is not supported by the data.

3. Data Analysis: Experiments 4, 5, and 6.

Part of the motivation for parameter set 2 used in experiments 4, 5, and 6 was the coefficient estimates obtained from (13) despite the fact that (13) turned out to be misspecified at closer inspection. Recall from Conclusion 3 that under the hypothesis of (13) the variable δ is so large that the adjustment process does not converge. The new parameters were chosen so that an asymptote exists for larger values of δ. Also, the Distinguished Nash equilibrium level is brought lower with

Table 5a: Test Statistics, Experiments 1, 2, 3

Exp. No.	Buyer	$t(\alpha)$	$t(\beta)$	F
1	1	−100.2754	50.8428	10374.7096
1	2	−20.3197	9.9512	978.0645
1	3	−73.7523	54.0459	968.6604
1	4	−71.3392	51.5514	602.9177
1	5	−68.4243	46.6207	1040.6981
1	6	−62.9264	42.0708	802.1850
1	7	−21.4292	4.9677	8591.5872
1	8	−30.8187	.7262*	2465.6702
1	9	−32.9490	6.9555	926.3306
1	10	−4.5447	1.2949*	2614.6547
1	11	−31.4043	7.5274	229.9863
1	12	−.5604*	.1497*	597.1841
2	1	−17.0421	15.5652	9.8651
2	2	−22.0391	20.4831	2.1224*
2	3	−18.7487	29.4693	15.4402
2	4	−35.6842	4.6281	10.2882
2	5	−9.8821	6.4088	5.6125*
2	6	7.1228	−23.4390	.1709*
2	7	−99.9535	71.6421	277.3643
2	8	−28.1058	49.8354	65.1687
2	9	−118.3948	65.7540	110.4960
2	10	−39.9502	66.2944	61.3589
2	11	−.0631*	−16.9363	2.2042*
2	12	.1285*	−.1300*	.0274*
3	1	−20.6041	44.9414	5.7791
3	2	−94.7928	185.0872	45.3135
3	3	−26.7493	40.1981	2.6523
3	4	−19.3087	35.0025	11.0152
3	5	−18.0584	19.3946	.7927*
3	6	−16.6774	30.1989	6.5742
3	7	−.3884*	.3778*	.0860*
3	8	−63.7093	19.0043	63.1033
3	9	−69.9173	77.1287	63.3463
3	10	−66.1234	36.3225	63.4482
3	11	−66.9559	69.0779	43.2158
3	12	−.3078*	.2976*	.1224*

* We cannot reject the corresponding null hypothesis at 1 percent level of significance.

Table 5b: Test Statistics, Experiments 4, 5, 6

Exp. No.	Buyer	$t(\alpha)$	$t(\beta)$	F
4	1	−19.8302	22.5847	101.0389
4	2	−114.4300	120.1893	277.9110
4	3	−31.8384	35.9962	148.5727
4	4	−153.1644	120.6668	434.6596
4	5	4.4767	−7.1621	9.6055
4	6	−30.9676	29.6616	17.9902
4	7	19.3038	−14.0446	80.1962
4	8	7.5263	−3.0604	58.9619
4	9	7.1722	−2.3303	93.0922
4	10	23.1449	−15.8084	113.8447
4	11	2.4017	−2.1940	1.1339*
4	12	−.8535*	−2.8092	19.2196
5	1	−79.3074	92.5125	95.3822
5	2	−50.7989	65.9981	72.4255
5	3	−77.8568	94.6752	105.2097
5	4	−27.6783	31.6808	14.8108
5	5	−6.5163	3.0467	.1095*
5	6	−40.3496	−42.0020	16.2343
5	7	14.0097	.8031*	1153.9850
5	8	9.4057	4.4122	502.4972
5	9	15.1050	−.4166	958.2708
5	10	12.9903	3.0896	2691.9955
5	11	15.3281	−4.4209	324.0271
5	12	−2.4175	1.4857*	6.5870
6	1	−20.9718	29.9024	65.5923
6	2	−46.5175	65.5258	108.9895
6	3	−40.7580	55.3101	35.1510
6	4	−29.4582	44.2471	59.2107
6	5	.4006*	−1.1408*	6.4283
6	6	−31.5853	29.9847	5.3108
6	7	−67.3839	88.6633	26.2216
6	8	−13.3079	20.8022	13.8950
6	9	−20.5770	29.5082	41.1271
6	10	−8.2781	11.3932	2.5077*
6	11	−.3328*	−.3106*	5.1790
6	12	11.1742	−13.0648	9.9007

* We cannot reject the corresponding null hypothesis at 1 percent level of significance.

the new parameters and subjects are recruited for longer time so that, given a rate of change of bids close to the one observed in the first set of experiments, the bids pass through the equilibrium levels earlier in the experimental session. This would leave more time to observe the possible continuations of the phenomena observed in the last several periods in experiments 2 and 3, e.g., increase in the variance of bids after the Distinguished Nash equilibrium levels are passed through, and the appearance of zero bids in the last several periods.

The graphs of actual bids for the experiments 4, 5, and 6 are in Figures 6 through 8. In all three experiments we observe convergence near the Distinguished Nash equilibrium in both markets. In spite of this observed convergence a surprising result is obtained from the econometric analysis.

Conclusion 7. The strategic linear adaptive model can be rejected.

Support: Table 4b contains the estimation and test results on model (14) for the new set of experiments. Tests on the coefficients from Table 5b reveal that only in three out of thirty-six cases do we fail to reject the null hypothesis that slope *and* intercept coefficients both have the values suggested by B2 and B3. Thus, the strategic linear adaptive model can be rejected.

Misspecification of model (13) is obvious from the fact that for more than half of the observations we have negative changes in the minimum accepted bids whereas the strategic linear adaptive model predicts a monotonic sequence of bids under Cournot expectations. So we do not estimate (13) with the new set of experiments. The econometric model reveals even deeper problems with this class of models. The next result shows that even the most basic form of the model can be rejected.

Conclusion 8. The general linear adaptive model can be rejected.

Support: The model does not survive the specification test. For about two-thirds of the cases the Godfrey statistics are too high at 1 percent significance level. At 10 percent significance level we fail to reject the null hypothesis for only seven out of thirty-six cases. (See Table 5b.)

Next, we discuss some features of the general linear adaptive model to investigate possible sources of error in the light of previous estimation and test results.

(i) The assumptions on expectation formation, C1, might have some justification in an asymptotic sense. Adaptive expectations can be justifiable from a rationality point of view in some particular kinds of stationary and nonstationary environments (Muth 1960). However, when applied to variables which exhibit trends or follow nonmonotonic paths, adaptive expectations imply persistent forecast errors. Asymptotically, as the underlying process becomes stationary these forecast errors go to zero, but in a finite sample which contains mainly disequilibrium observations persistent under- and over-prediction is a serious problem. In the same way, assumption C2 has an asymptotic justification at best. That is, in assuming that everybody has the same λ we are essentially arguing that if the expectations of each bidder are to be fulfilled in the limit then in the limit everybody must have the same adjustment parameter λ. Another problem with this expectation formation model is that the agent's decision affects the variable on which he/she forms his/her expectation. However, since incorporation of this last remark in a model of adjustment process would require a full game-theoretic analysis of the underlying repeated game, we do not pursue this here.

(ii) Despite the possible shortcomings of the expectation formation assumptions discussed above, we feel that the real problem is with the behavioral assumptions of the previous section. The strategic linear adaptive model (Model 2) when combined with assumption C2, has a special relation to the static Nash equilibrium: the conjunction of the assumptions imply that aspects of the static Nash equilibrium (zero profit for the marginals, inframarginals bidding "just above" the marginal's bid, linearity of the marginal bid in revenue) hold in every period during the adjustment process. However, inspection of the estimates for λ_i in Table 5a reveal that under the maintained hypotheses B1 and C1, and the statistical assumptions of u_{it} (which by Conclusion 4 are justified for at least the first set of experiments), *none* of the thirty-six cases in experiments 1, 2, and 3 satisfy C2, B2, and B3 at the same time.

Generally, the behavioral assumptions lack aesthetic and theoretical appeal from a process modeling point of view as will be discussed next.

(iii) The only merit of B1 from a theoretical point of view is that it subsumes as a special case the behavior suggested by B2 and B3. We would like to have a decision rule which constitutes a solution to the decision-making problem facing the agent. B1 lacks this property.

(iv) B1 through B3 use aspects of the one-shot Nash equilibrium and they are what we call system-consistency conditions. Notice that the identification of marginal and inframarginal bidders is accomplished by an application of equilibrium conditions. The zero-profit bidding strategy for the marginal bidders, inframarginals bidding "just above" the marginals' bid and linearity of the marginals' bid, are all features of the one-shot Nash equilibrium which we should not expect to be true out of equilibrium. For instance, out of equilibrium the marginal bidders need not bid so as to make their profits zero; rather they would have expectations about the minimum accepted bid in the next period in addition to their expectations about the revenue in the next period, and tender a bid which would give them positive expected profits as long as their expectation of the minimum accepted bid in the next period is less than the bid which would make their profits zero. That is, we can expect B2 and B3 to hold only asymptotically, not in every period.

In the next section we drop the traditional approach based on system-wide consistency conditions and develop models which we believe do not suffer from the problems discussed above.

B. *An Alternative Adjustment Process*:

The models we present in this section are based on decision-theoretic principles and do not impose the equilibrium properties of a game as conditions to be satisfied every period. We assume that the decision problem facing each bidder in every period is one of expected profit maximization under uncertainty. We leave for future research the possibility of modeling the situation as a repeated game with incomplete information. Here, we seek to investigate the one-shot NE concept as the stationary point of a process. We have developed models that require less "rationality" on the part of the players since the steady state interpretation of the Nash equilibrium does not involve the "hyper rationality" of immediate equilibrium behavior that is imposed on the model by a correct game theoretic analysis.

D1. At each period t, every bidder i, $i = 1, 2, \dots, 12$, is assumed to consider only *three* statistics of the behavior of other bidders in deciding how much to bid:

Z_{it} = the rebate factor for bidder i at time t,
Y_{it} = second lowest bid among five rival bids in bidder i's market, and
$\tilde{R}_{it}$ = revenue from the three highest bidders among bidder i's five rivals in the market in which i belongs, *plus* revenue from the other market.

E1. Bidder i believes that $(Z_{it}, \tilde{R}_{it}, Y_{it})$ is a *random* vector distributed according to a law the joint density of which is $g_{it}(Z_{it}, \tilde{R}_{it}, Y_{it})$.

We assume that i's beliefs about others' beliefs, about others' beliefs about i's belief . . . , etc., do not play a role in his decision-making, or they are all summarized in $g_{it}(\cdot,\cdot,\cdot)$ together with everything i knows about the structure of the game he/she is in. We do not go into details of how beliefs about the underlying unknown variables, i.e., the valuations of the other bidders, are transformed into beliefs about the three statistics Z_{it}, $\tilde{R}_{it}$, and Y_{it}. For our purposes, g_{it} might have been formed taking into account the other bidders strategies in a fully "rational" way or might be a joint distribution without such game-theoretic content. For the specific cases considered below, however, we use the latter interpretation since the former would result in a host of additional questions which cannot be dealt with without a full-blown game-theoretic model.

D2. Each buyer i is a myopic expected profit maximizer.
Bidder i's profit at time t is:

$$\Pi_{it} = [V_i + Z_{it}\tilde{R}_{it} + (Z_{it} - 1)B_{it}]1_{\{Y_{it} < B_{it}\}} \tag{15}$$

where $1_{\{Y_{it} < B_{it}\}} = 1$ if $Y_{it} < B_{it}$, 0 otherwise. Thus, in each period bidder i chooses his bid, B so as to maximize

$$E\Pi_{it} = \int_{\{Y_{it} < B_{it}\}} [V_i + Z_{it}\tilde{R}_{it} + (Z_{it} - 1)B_{it}]dg_{it}(Z_{it}, \tilde{R}_{it}, Y_{it}). \tag{16}$$

Let $E_*(\cdot)$ denote the conditional expectation of a random variable given that $Y_{it} < B_{it}$ and, let $F_{it}(\cdot)$ be the marginal distribution of Y_{it}. Then (16) becomes

$$E\Pi_{it} = [V_i + E_*(Z_{it} \cdot \tilde{R}_{it}) + (E_*(Z_{it}) - 1)B_{it}]F_{it}(B_{it}). \tag{17}$$

Under D1, D2, and E1, i's bid satisfies the first order condition for all t:

$$F_{it}'(B_{it})[V_i + E_*(Z_{it} \cdot \tilde{R}_{it}) + (E_*(Z_{it}) - 1)B_{it}] + F_{it}(B_{it})[E_*(Z_{it}) - 1 + E_*'(Z_{it}) + E_*'(Z_{it} \cdot \tilde{R}_{it}) = 0 \tag{18}$$

where F_{it}' and $E_*'(\cdot)$ are the derivatives with respect to B_{it} of F_{it} and $E_*'(\cdot)$, respectively.

Without some further structure on $g_{it}(\cdot,\cdot,\cdot)$, (18) is without any important content because any observed behavior can be explained as a solution to an expected utility maximization problem for some belief structure. In particular, to compute $E_*(Z_{it} \cdot \tilde{R}_{it})$, $E_*(Z_{it})$, $E_*(\tilde{R}_{it})$ and $F_{it}(B_{it})$ we need to specify the joint density $g_{it}(\cdot,\cdot,\cdot)$, possibly up to some unknown parameters. Moreover, we need to specify how the beliefs $g_{it}(\cdot,\cdot,\cdot)$ are upgraded given the past history.

The following special cases are possible and simple candidates. The first case, E2, is natural but as will be discussed, it is not tractable given the current state of theory. The second case, E3, involves further restrictions which might be undesirable but it yields an operational model.

E2. Assume that $(\tilde{R}_{it}, Y_{it})$ is jointly normal and Z_{it} is independent of $(\tilde{R}_{it}, Y_{it})$ with mean $\bar{Z}_{it}$:

$$\begin{bmatrix} \tilde{R}_{it} \\ Y_{it} \end{bmatrix} \sim N\left[\begin{bmatrix} \bar{\tilde{R}}_{it} \\ \bar{Y}_{it} \end{bmatrix}, \begin{bmatrix} \sigma_{\tilde{R}_{it}} & \rho_{it} \\ \rho_{it} & \sigma_{Y_{it}} \end{bmatrix}\right].$$

In this case

$$E_*(\tilde{R}_{it}) = \bar{\tilde{R}}_{it} - \rho_{it}\sigma_{R_{it}} \frac{\phi\left(\dfrac{B_{it} - \bar{Y}_{it}}{\sigma_{Y_{it}}}\right)}{\Phi\left(\dfrac{B_{it} - \bar{Y}_{it}}{\sigma_{Y_{it}}}\right)}$$

and the first order conditions (18) becomes

$$B_{it} + \sigma_{Y_{it}} \frac{\Phi[(B_{it} - \bar{Y}_{it})/\sigma_{Y_{it}}]}{\phi[(B_{it} - \bar{Y}_{it})/\sigma_{Y_{it}}]} = \frac{V_i}{1 - \bar{Z}_{it}} + \frac{\bar{Z}_{it}}{1 - \bar{Z}_{it}}[\bar{\tilde{R}}_{it} + \rho_{it} \frac{\sigma_{\tilde{R}_{it}}}{\sigma_{Y_{it}}} (\bar{Y}_{it} - B_{it})] \tag{19}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and *cdf* of standard normal, respectively.

To specify how the beliefs, $g_{it}(\cdot,\cdot,\cdot)$ are updated over time, it now suffices to specify how $(\bar{Z}_{it}, \bar{\tilde{R}}_{it}, \bar{Y}_{it}, \sigma_{\tilde{R}_{it}}, \sigma_{Y_{it}}, \rho_{it})$ are determined, given the past. In one attempt to estimate (19) we assumed that $\bar{Y}_{it}$ is the minimum mean square error forecast of Y_{it} based on observations on Y_{it} up to time $t-1$ in a model which specified Y_{it} as a linear function of its lagged value. The standard error of forecast from this prediction rule is taken to be $\sigma_{Y_{it}}$. In this attempt, the same was done to generate proxies for $\bar{\tilde{R}}_{it}$ and $\sigma_{\tilde{R}_{it}}$. ρ_{it} was taken to be the sample correlation between $\tilde{R}_{it}$ and Y_{it} up to time t and $\bar{Z}_{it} = Z_i'$ for all t. However, with the beliefs upgraded in this way, serious numerical problems appear because, based on the estimated $\sigma_{Y_{it}}$, the function $\phi(\cdot)$ is essentially zero for most of the sample. Beliefs generated on the basis of forecast models which involve two-period lagged values of the relevant variables also did not solve this problem. In addition, forecasts generated by these models exhibit persistent over- and under-prediction. Thus we dropped E2 from further consideration.

Next we consider axioms placing further restrictions on the resulting model and which involve fewer unobserved variables. The axioms dictate the form of $g_{it}(\cdot,\cdot,\cdot)$ and the nature of updating over time. Two cases are considered. In one case the updating is based on lagged values and in the other case expectations are more forward looking.

E3. Assume

(i) $Z_{it}, \tilde{R}_{it}$ and Y_{it} are independently distributed with means $\bar{Z}_{it}, \bar{\tilde{R}}_{it}$, and $\bar{Y}_{it}$;

(ii) Y_{it} is uniform on $[\bar{Y}_{it} - \Delta_{it}, \bar{Y}_{it} + \Delta_{it}]$;

(iii) $\bar{\tilde{R}}_{it} = R_{i,t-1}$;

(iv) $\bar{Y}_{it} = Y_{i,t-1}$;

(v) $\bar{Z}_{it} = \dfrac{X_i}{\Sigma_{j \in W_{t-1}} X_j}$;

(vi) $\Delta_{it} = \rho_i^t \Delta_{i0}$.

E4. Assume

(i) $Z_{it}, \tilde{R}_{it}$, and Y_{it} are independent with means $\bar{Z}_{it}, \bar{\tilde{R}}_{it}$, and $\bar{Y}_{it}$;

(ii) Y_{it} is uniform on $[\bar{Y}_{it} - \Delta_{it}, \bar{Y}_{it} + \Delta_{it}]$;

(iii) $\bar{\tilde{R}}_{it} = R_{it}$;

(iv) $\bar{Y}_{it} = Y_{it}$;

(v) $\bar{Z}_{it} = \dfrac{X_i}{\Sigma_{j \in W_t} X_j}$;

(vi) $\Delta_{it} = \rho_i^t \Delta_{i0}$.

Observe that E3(i)(ii) and E4(i)(ii) are the same assumptions. Under either assumption E3 or E4 the maximization problem (16) can be solved. The optimum bid, the solution to (18) when the corner solutions are considered, is given by:

$$B_{it} = \begin{cases} \bar{Y}_{it} - \Delta_{it} & \text{if } \bar{\bar{R}}_{it} \leq \dfrac{(\bar{Y}_{it} - \Delta_{it})(1 - \bar{Z}_{it}) - V_i}{Z_i} \\ \bar{Y}_{it} + \Delta_{it} & \text{if } \bar{\bar{R}}_{it} \geq \dfrac{(\bar{Y}_{it} + 3\Delta_{it})(1 - \bar{Z}_{it}) - V_i}{\bar{Z}_{it}} \\ \dfrac{V_i}{2(1 - \bar{Z}_{it})} + \dfrac{\bar{Z}_{it}}{2(1 - \bar{Z}_{it})} \bar{\bar{R}}_{it} + \dfrac{1}{2} \bar{Y}_{it} - \dfrac{1}{2} \Delta_{it} & \textit{otherwise} \end{cases} \tag{20}$$

As a matter of fact (20) gives the unique solution to the maximization of (16) unless the first condition in (20) holds, in which case there are many solutions which all give zero expected profit and are characterized by the condition $B_{it} \leq \bar{Y}_{it} - \Delta_{it}$. Thus, the particular solution (20) has the property of being the largest of these optimal bids.

From E3 and E4 together with previous assumptions we obtain two new and different models.

Model 3. The *lagged expectation decision theoretic model* consists of assumptions D1, D2, E1, and E3.

Model 4. The *unbiased expectations decision-theoretic model* consists of assumptions D1, D2, E1, and E4.

The term "unbiased" is used because on average, there will be no difference between the mean of the beliefs about the value of a variable and the actual mean of the random variable. The expectations are not based on perfect foresight because beliefs still have a variance. Expectations are not fully rational because all data are not used to generate the estimate. Furthermore, the beliefs cannot be rationalized because the individual behavior based on those beliefs does not generate system data consistent with those beliefs. In fact, as will be seen below, the beliefs as postulated in Model 4 can have significant aspects of irrationality.

Both Models 3 and 4 are closely related to the static Nash equilibrium as is demonstrated by the following proposition.

Proposition 3. If $\rho_i = \rho$ for every i and if $0 < \rho < 1$, then the bid functions implied by Model 3 and Model 4 asymptotically approach a Nash equilibrium bid function.

Proof. Δ approaches zero as t approaches infinity and (20) becomes a Nash equilibrium bid function. Then notice that the marginal agents' behavior is asymptotically given by the first condition of (20) and the inframarginal agents are asymptotically given by the second condition.

For every individual, the parameters in Models 3 and 4 are $(\rho_i, \Delta_{i0}, V_i)$. Although possible in principle, estimation of these parameters is complicated and time-consuming. Moreover, suggested procedures for similar but less complex models with switching regimes give estimates, but the sampling properties of which are, in general, unknown (Judge et al. 1980). To reduce the dimensionality of the problem, we take advantage of the fact that V_i are known coefficients. To estimate the remaining parameters, we apply nonlinear least squares technique to (20) (see, e.g., White and Domowitz 1984). Specifically, we minimize the sum of squared errors using a two-dimensional grid search for ρ_i and Δ_{i0} to obtain estimates $\hat{\rho}_{i0}$, $\hat{\Delta}_{i0}$ of the remaining coefficients.

To evaluate Models 3 and 4 we look at the forecast accuracy of these models. The estimates for ρ_i and Δ_{i0} for each individual using the first $T-8$ observations in the sample are given in Tables 6 and 7 for Models 3 and 4 respectively. The R^2s for the estimation periods are reported in Table 8. Using the estimated coefficients, we generate forecasts for the last eight observations and compare them to the observed bids. Lastly, instead of performing a detailed specification testing of Models 3 and 4 as we did for the general linear adaptive model, we compare the forecast performance of these models to the models presented in the previous section, Table 9, Table 10 and Table 11.

Conclusion 9. Among the four, Model 2 is decidedly worse than the other three models. Model 1 and Model 3 cannot be distinguished in view of the statistical evidence. Model 4 is the best from among the four models.

Support: Tables 9 and 10 contain statistics for comparing forecast performance of four models when applied to the time series of bids for each of the 72 individuals in the six experiments. Each model is estimated using the initial $T-8$ bids for each individual and then forecasts for the last eight bids are obtained based on the estimated parameter values. The forecast accuracy measures we use are root-mean-squared error (*RMSE*) and Theil's U coefficient. For both measures, lower values are associated with better forecasts:

$$RMSE = \left[\sum_{t=T-7}^{T} (\hat{x}_t - x_t)^2 / 8 \right]^{1/2}$$

$$U = \frac{\left[\sum_{t=T-7}^{T} (\hat{x}_t - x_t)^2 / 8 \right]^{1/2}}{\left[\sum_{t=T-7}^{T} \hat{x}_t^2 \right]^{1/2} + \left[\sum_{t=T-7}^{T} x_t^2 \right]^{1/2}}$$

where x_t is the actual value of the variable to be predicted and $\hat{x}_t$ the predicted value.

The statistics are presented in Tables 9 and 10. Table 11 contains a summary of the comparison.[8] Model 1 has the lowest *RMSE* in twenty out of seventy-two cases, Model 2 in fifteen cases, Model 3 in eleven cases, and Model 4 in twenty-seven cases when all four models are considered for choice. Theil's U gives the same ordering when the four models are considered simultaneously. Although Model 4 is the best model in all subsets of the four models, ranking of Model 3 improves considerably when the four models are ranked based on pairwise *RMSE*

Table 6: Parameter Estimates for Model 3
(Based on $T - 8$ observations)

Exp. No.	Subject	$\hat{\rho}$	$\hat{\Delta}_0$	Exp. No.	Subject	$\hat{\rho}$	$\hat{\Delta}_0$
1	1	.75	36	4	1	1.00	13
1	2	.51	144	4	2	1.02	10
1	3	.99	66	4	3	1.02	10
1	4	.97	82	4	4	.94	13
1	5	.85	19	4	5	1.02	51
1	6	.96	6	4	6	.94	20
1	7	.69	24	4	7	.95	23
1	8	1.01	121	4	8	1.01	74
1	9	.99	65	4	9	1.00	55
1	10	.73	18	4	10	.96	19
1	11	.97	37	4	11	1.02	4
1	12	1.02	18	4	12	1.01	9
2	1	.99	128	5	1	1.00	139
2	2	.98	138	5	2	1.01	100
2	3	.93	69	5	3	1.00	100
2	4	.96	58	5	4	.98	60
2	5	1.08	4	5	5	.99	38
2	6	.96	23	5	6	1.15	5
2	7	1.01	127	5	7	.98	9
2	8	1.01	124	5	8	.87	27
2	9	1.07	6	5	9	.97	10
2	10	1.13	3	5	10	.93	16
2	11	.82	250	5	11	.52	6
2	12	.97	18	5	12	1.05	6
3	1	.98	127	6	1	.99	15
3	2	.99	110	6	2	1.00	15
3	3	1.06	6	6	3	.98	13
3	4	1.03	8	6	4	.95	31
3	5	.87	63	6	5	1.03	31
3	6	.88	58	6	6	1.06	4
3	7	1.01	123	6	7	.95	118
3	8	1.15	2	6	8	1.08	5
3	9	.96	45	6	9	.99	69
3	10	.96	51	6	10	1.04	11
3	11	.92	37	6	11	1.03	16
3	12	.92	35	6	12	1.02	31

Table 7: Parameter Estimates for Model 4
(Based on $T - 8$ observations)

Exp. No.	Subject	$\hat{\rho}$	$\hat{\Delta}_0$	Exp. No.	Subject	$\hat{\rho}$	$\hat{\Delta}_0$
1	1	.64	63	4	1	1.02	8
1	2	.55	250	4	2	1.06	4
1	3	.52	68	4	3	.98	161
1	4	.98	85	4	4	.59	44
1	5	.52	170	4	5	1.01	63
1	6	.51	3	4	6	.91	48
1	7	.68	41	4	7	.97	17
1	8	.52	66	4	8	1.00	88
1	9	.99	70	4	9	1.05	3
1	10	.51	116	4	10	.97	18
1	11	.98	40	4	11	.51	125
1	12	.83	16	4	12	.96	24
2	1	.95	5	5	1	1.00	145
2	2	1.09	1	5	2	1.00	123
2	3	.96	87	5	3	.99	125
2	4	.52	161	5	4	.96	86
2	5	.93	81	5	5	.98	50
2	6	1.04	21	5	6	1.13	6
2	7	.51	95	5	7	.51	64
2	8	.56	123	5	8	.92	17
2	9	1.01	44	5	9	1.01	4
2	10	1.12	1	5	10	.92	16
2	11	.87	250	5	11	1.04	1
2	12	.57	71	5	12	1.01	16
3	1	1.01	114	6	1	1.02	7
3	2	.65	78	6	2	1.02	9
3	3	1.01	1	6	3	1.02	5
3	4	.51	35	6	4	1.06	2
3	5	.62	212	6	5	1.02	39
3	6	1.08	1	6	6	1.07	2
3	7	.93	6	6	7	.99	93
3	8	.51	60	6	8	.99	91
3	9	.74	33	6	9	.98	89
3	10	.67	38	6	10	1.00	31
3	11	.51	1	6	11	1.02	20
3	12	.97	1	6	12	1.02	30

Table 8: Estimation Period R^2

Exp. No.	Subject	Model 1	Model 2	Model 3	Model 4
1	1	.9939	.6134	.9662	.9877
1	2	.9740	.5168	.9040	.8356
1	3	.9926	.5197	.9826	.9913
1	4	.9936	.4648	.9808	.9503
1	5	.9876	.5963	.9709	.9517
1	6	.9922	.4970	.9819	.9591
1	7	.9932	.1690	.8729	.9426
1	8	.9614	.7651	.8785	.7773
1	9	.9848	.7268	.9697	.9484
1	10	.9918	.8186	.8897	.9042
1	11	.9523	.7900	.9399	.9150
1	12	.9524	.8363	.7930	.7938
2	1	.9938	.0493	.9872	.9894
2	2	.9782	.0750	.9663	.9756
2	3	.9964	.0184	.9789	.9892
2	4	.9924	.0145	.9594	.9865
2	5	.9938	.0421	.9766	.9869
2	6	.9849	.0010	.9235	.9156
2	7	.9933	.1106	.9684	.9699
2	8	.9755	−.0136	.8957	.9735
2	9	.8974	−.0098	.8070	.8614
2	10	.9629	.0459	.9148	.9609
2	11	.8656	.0120	.8854	.8709
2	12	.8818	.0378	.8992	.9743
3	1	.9958	−.0350	.9950	.9919
3	2	.9981	−.0748	.9952	.9862
3	3	.9976	−.0330	.9902	.9975
3	4	.9986	−.0171	.9892	.9975
3	5	.9985	−.0262	.9769	.9905
3	6	.9940	−.0252	.9665	.9939
3	7	.9881	.0432	.9808	.9872
3	8	.9883	.0701	.9872	.9783
3	9	.9923	−.0447	.9480	.8747
3	10	.9875	.0236	.9599	.9697
3	11	.9986	.0992	.9863	.9956
3	12	.9947	.0891	.9749	.9947

Table 8 (*Continued*)

Exp. No.	Subject	Model 1	Model 2	Model 3	Model 4
4	1	.9784	−.0708	.9608	.9353
4	2	.9755	−.0478	.9560	.9424
4	3	.9843	−.0521	.9712	.9524
4	4	.9725	−.0207	.9814	.9621
4	5	.0324	.0104	−.1715	−.1544
4	6	.9760	−.0095	.9766	.9793
4	7	.7127	−3.2091	.5076	.3579
4	8	.9408	−3.5103	.8687	.8378
4	9	.9546	−2.3849	.8548	.8317
4	10	.8253	−5.7954	−1.5861	−1.8586
4	11	.9248	−.1287	.9122	.5455
4	12	.8999	.0565	.8636	.7462
5	1	.9609	−.2432	.7451	.7034
5	2	.9368	−.5596	.6808	.6898
5	3	.9590	−.3289	.6724	.6961
5	4	.9188	−.0352	.3015	.2703
5	5	.2964	.0004	.1749	.1660
5	6	.0871	.0112	.4091	.3419
5	7	.9815	−5.6413	.9613	.8519
5	8	.9548	−6.3565	.8141	.8820
5	9	.9844	−6.6958	.9583	.9575
5	10	.9934	−7.5376	.9262	.8096
5	11	.9591	−4.0174	.9088	.9246
5	12	.9361	−.0250	.8394	.7461
6	1	.9584	−.7548	.9474	.8966
6	2	.9614	−.8790	.9209	.8490
6	3	.9176	−.4275	.9337	.9008
6	4	.9543	−.8167	.9714	.9377
6	5	.1214	−.0005	−.0820	−.0378
6	6	.8940	−.0046	.9180	.9259
6	7	.7181	−.2524	.7839	.5927
6	8	.6173	−.2238	.7591	.4857
6	9	.8506	−.3609	.7830	.5873
6	10	.5829	−.0988	.4327	.3124
6	11	.2604	.0142	.0870	.3923
6	12	.2132	.0002	−.5414	−.3396

Table 9: Root Mean Squared Error

Exp. No.	Subject	Model 1	Model 2	Model 3	Model 4
1	1	5.0044	12.7610	8.6946	1.9673
1	2	2.8807	12.6738	8.7105	2.1503
1	3	10.1648	11.9177	9.3358	2.3452
1	4	7.9601	12.8136	6.2096	5.4050
1	5	7.5896	13.1149	7.0743	1.5811
1	6	6.3157	13.4094	4.7668	1.8028
1	7	1.4616	23.1965	3.8301	1.4522
1	8	4.5522	22.9423	2.2667	1.4577
1	9	1.3308	24.8199	2.0921	3.4555
1	10	10.1908	23.8264	3.2601	1.1726
1	11	1.9656	23.1151	3.0118	2.9773
1	12	13.3200	22.3341	2.8726	2.0866
2	1	43.8600	53.8555	15.9549	34.6988
2	2	46.4090	54.4100	17.4352	27.8765
2	3	68.7419	48.2436	47.7514	24.7200
2	4	70.5841	34.3853	35.6305	14.4307
2	5	207.9288	16.4528	22.3368	18.7806
2	6	10.2511	167.4522	167.0524	164.4814
2	7	86.5671	47.0971	75.2085	177.7881
2	8	114.3237	43.0453	77.5901	176.6931
2	9	12.1542	42.5655	124.5541	147.9618
2	10	32.1716	47.7582	120.0194	172.2621
2	11	296.4577	148.0377	158.1495	169.8515
2	12	261.5190	215.4075	221.5227	252.5885
3	1	77.6482	79.4317	45.6869	42.7483
3	2	77.4096	91.2905	39.4562	63.2944
3	3	73.7920	51.2777	45.2817	51.4630
3	4	80.7749	61.7365	48.9196	36.7916
3	5	76.6143	33.9323	26.3032	27.0275
3	6	69.6497	52.7505	48.2757	22.9148
3	7	25.3851	33.3181	24.6253	20.5813
3	8	23.8118	27.1221	23.4605	11.9687
3	9	33.2069	19.2309	15.0360	10.1113
3	10	25.2690	32.9230	26.9231	16.5441
3	11	20.2741	19.1870	24.4982	16.7929
3	12	20.3638	36.9389	33.5408	25.4352

Table 9 (*Continued*)

Exp. No.	Subject	Model 1	Model 2	Model 3	Model 4
4	1	1.5148	8.6874	8.7106	9.3753
4	2	2.9547	10.8033	9.7832	15.1822
4	3	1.7737	8.3463	10.3916	11.2762
4	4	9.9634	10.4070	10.9883	10.9886
4	5	69.3824	91.4544	67.4737	67.7670
4	6	8.7056	10.7873	10.3054	13.3665
4	7	5.2204	6.9427	2.1898	1.6835
4	8	5.3590	3.4524	4.9418	1.5837
4	9	6.4366	6.0883	2.2810	9.9436
4	10	6.8175	3.7276	2.0841	2.9034
4	11	1.1362	.6286	2.0729	3.3541
4	12	4.0743	6.1775	2.7923	4.5150
5	1	7.1973	4.7970	12.8779	12.5973
5	2	15.1496	6.7379	9.6075	20.2853
5	3	9.7026	7.0888	14.4640	23.1782
5	4	4.4915	5.2953	6.2651	2.2504
5	5	5.9228	1.9856	12.2709	10.7712
5	6	100.7203	140.8128	.0000	.0000
5	7	1.2810	20.4035	1.6265	4.4581
5	8	3.7056	19.1396	4.0858	2.9594
5	9	2.4874	21.8270	2.9917	1.3246
5	10	1.4674	19.6518	2.7232	3.1274
5	11	1.5111	16.0300	3.2016	2.6583
5	12	.7855	1.4618	6.5088	1.9405
6	1	8.9980	2.8347	5.5694	11.6816
6	2	9.0486	3.5806	8.8600	14.4587
6	3	3.8866	2.3237	3.9362	6.8210
6	4	4.9596	3.3339	3.5361	3.2135
6	5	13.3544	18.9020	37.7629	31.7459
6	6	7.7271	8.6638	13.8256	10.4179
6	7	3.7875	7.8301	9.3748	10.8550
6	8	5.9905	12.1778	20.2985	7.9448
6	9	3.8408	8.6211	2.7324	8.3689
6	10	28.5964	23.5592	24.2169	23.7142
6	11	12.9072	14.0228	18.7723	16.4953
6	12	3.4188	1.4892	32.9025	32.9052

Table 10: Theil's U Coefficient

Exp. No.	Subject	Model 1	Model 2	Model 3	Model 4
1	1	.0193	.0466	.0331	.0073
1	2	.0110	.0463	.0332	.0080
1	3	.0397	.0433	.0354	.0087
1	4	.0309	.0468	.0231	.0200
1	5	.0298	.0480	.0270	.0059
1	6	.0248	.0491	.0181	.0067
1	7	.0113	.1530	.0300	.0111
1	8	.0341	.1501	.0172	.0112
1	9	.0107	.1655	.0161	.0263
1	10	.0743	.1576	.0256	.0090
1	11	.0154	.1525	.0224	.0221
1	12	.0930	.1461	.0220	.0159
2	1	.0544	.0749	.0200	.0444
2	2	.0572	.0756	.0217	.0354
2	3	.0838	.0673	.0633	.0318
2	4	.0899	.0488	.0483	.0190
2	5	.2690	.0239	.0314	.0252
2	6	.0200	.2382	.2420	.2337
2	7	.1110	.0995	.1483	.3759
2	8	.1862	.0920	.1538	.3770
2	9	.0240	.0916	.2619	.3235
2	10	.0585	.1007	.2516	.3601
2	11	.4798	.3395	.3606	.3678
2	12	.6906	.6490	.6811	.7022
3	1	.0859	.0983	.0505	.0472
3	2	.0828	.1108	.0430	.0715
3	3	.0854	.0660	.0551	.0609
3	4	.0909	.0777	.0586	.0430
3	5	.0892	.0442	.0324	.0322
3	6	.0796	.0670	.0587	.0270
3	7	.0490	.0678	.0441	.0383
3	8	.0445	.0550	.0417	.0225
3	9	.0614	.0390	.0291	.0190
3	10	.0456	.0657	.0509	.0307
3	11	.0388	.0389	.0483	.0317
3	12	.0389	.0751	.0644	.0471

Table 10 (*Continued*)

Exp. No.	Subject	Model 1	Model 2	Model 3	Model 4
4	1	.0056	.0323	.0316	.0338
4	2	.0106	.0399	.0345	.0520
4	3	.0064	.0311	.0370	.0395
4	4	.0388	.0405	.0427	.0432
4	5	.3794	.4116	.3886	.3843
4	6	.0346	.0423	.0411	.0534
4	7	.0350	.0503	.0154	.0118
4	8	.0375	.0257	.0373	.0114
4	9	.0431	.0444	.0158	.0656
4	10	.0470	.0277	.0149	.0207
4	11	.0090	.0048	.0162	.0252
4	12	.0330	.0490	.0232	.0368
5	1	.0242	.0167	.0424	.0417
5	2	.0488	.0233	.0316	.0644
5	3	.0319	.0245	.0467	.0730
5	4	.0155	.0185	.0219	.0077
5	5	.0213	.0070	.0452	.0395
5	6	1.0000	1.0000	.0000	.0000
5	7	.0070	.1253	.0089	.0250
5	8	.0202	.1185	.0231	.0167
5	9	.0135	.1328	.0163	.0071
5	10	.0081	.1214	.0152	.0176
5	11	.0087	.1015	.0181	.0155
5	12	.0057	.0104	.0489	.0141
6	1	.0344	.0112	.0216	.0443
6	2	.0343	.0141	.0335	.0536
6	3	.0150	.0091	.0154	.0261
6	4	.0190	.0131	.0139	.0126
6	5	.0605	.0790	.1997	.1627
6	6	.0321	.0358	.0612	.0454
6	7	.0270	.0582	.0636	.0716
6	8	.0409	.0869	.1518	.0507
6	9	.0271	.0630	.0188	.0558
6	10	.2175	.1840	.2086	.1868
6	11	.1201	.1173	.1959	.1665
6	12	.0280	.0118	.3528	.3528

Table 11a: The Number of Agents for Which the Model Yields the Lowest Root Mean Square Error from Among the Comparison Models

RMSE				
Comparison Models	Model 1	Model 2	Model 3	Model 4
1, 2, 3, 4	20	15	11	27
1, 2, 3	27	19	26	
1, 2, 4	22	17		33
1, 3, 4	26		18	29
2, 3, 4		25	16	32
1, 2	45	27		
1, 3	34		38	
1, 4	34			38
2, 3		31	41	
2, 4		29		43
3, 4			32	39

Table 11b: The Number of Agents for Which the Model Yields the Lowest Theil's U from Among the Comparison Models

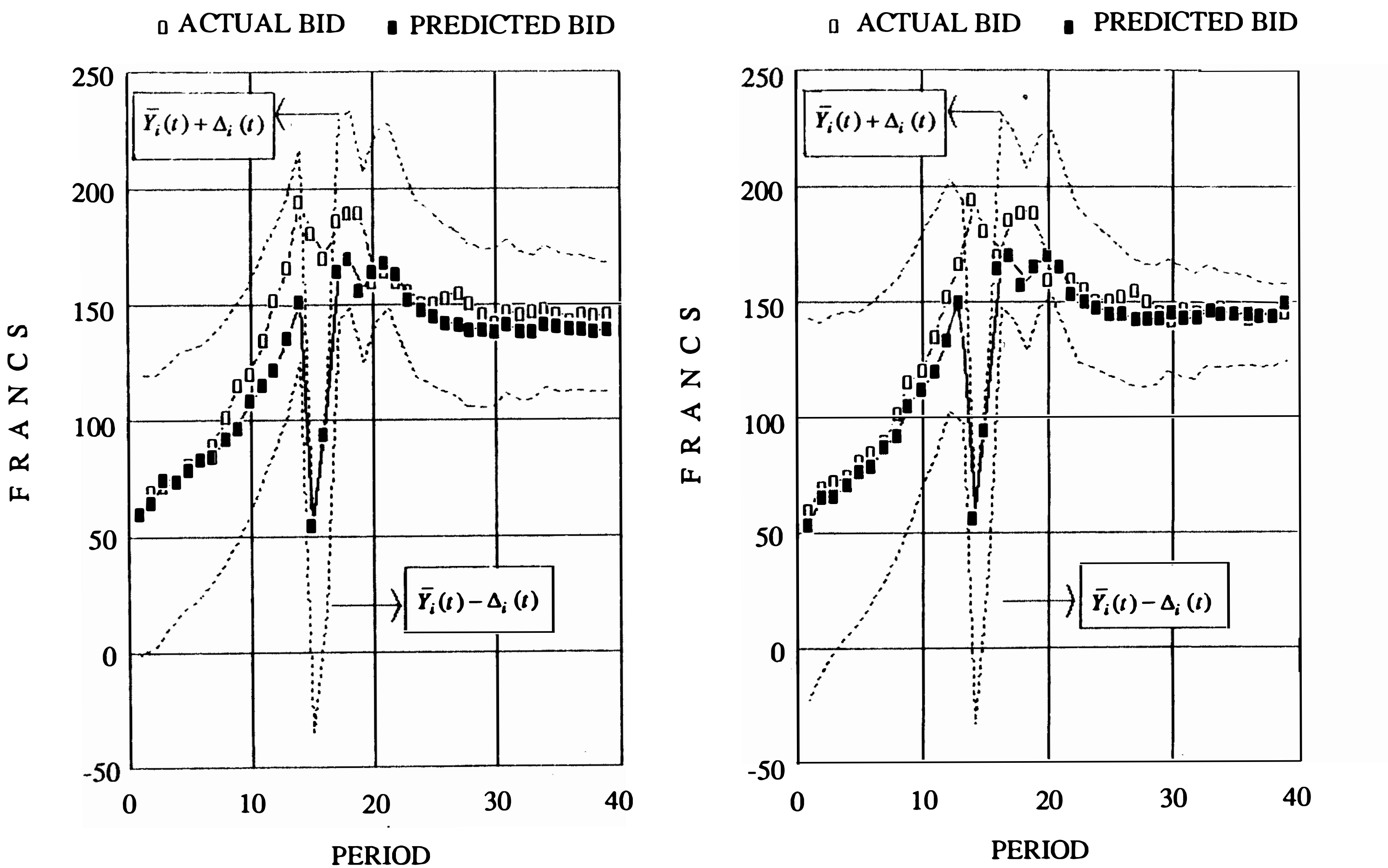
THEIL'S U				
Comparison Models	Model 1	Model 2	Model 3	Model 4
1, 2, 3, 4	20	16	9	28
1, 2, 3	28	18	26	
1, 2, 4	23	16		33
1, 3, 4	28		15	30
2, 3, 4		24	15	34
1, 2	45	26		
1, 3	36		36	
1, 4	36			36
2, 3		30	42	
2, 4		28		44
3, 4			29	41

comparisons; according to pairwise comparisons the four models are ranked as Model 4, Model 3, Model 1, and Model 2 from best to worst in terms of *RMSE*. Pairwise comparisons based on Theil's U do not give a unique transitive ordering of the four models. However, Model 4 is at least as good as the competing models in all subsets of the four models in terms of Theil's U.

An argument based on aesthetics can be made for a choice of Model 3 or 4 over Model 1. On the surface the improvement of Model 3 or 4 over Model 1 seems not overwhelming and therefore any choice of Model 3 or 4 over Model 1 must involve some intuition beyond those embodied in the statistics. The intuition begins with the observation that the extremely high R^2 of Model 1 suggest that little leeway exists for any improvement at all on Model 1. Nevertheless, Models 3 and 4 do achieve some improvement in a forecasting sense and they do so by utilizing only two parameters to be estimated (ρ, Δ_0) as opposed to the three parameters of Model 1 (α, β, λ). Furthermore, we know from Conclusion 8 that Model 1 fails all specification tests for the second set of experiments. Even though the same tests cannot be perfonned on Models 3 and 4 they could do no worse than Model 1.

We refrain from calling Model 4 a rational expectations model because of the obvious irrationality implicit in the constituent assumptions except E4.iii, iv, and v. Even when one is willing to maintain all the assumptions and impose the restriction that $0 < \rho_i < 1$, the beliefs may assign positive probability to impossible events; e.g., although negative bids are impossible in any period, Δ_{i0} may take values which, for some periods, imply a positive probability on the impossible event that Y_{it} takes a negative value. Figure 9 contains an example where this occurs based on the estimated parameters. In experiment 5, subject 4 exhibits the behavior between periods 10 and 20. In the case of Model 3, another check for a violation of rationality may be done by looking at the situations where the observed value of Y_{it} falls outside the internal $[\bar{Y}_{it} - \Delta_{it}, \bar{Y}_{it} + \Delta_{it}] =$ $[Y_{i,t-1} - \rho_i^t \Delta_{i0}, Y_{i,t-1} + \rho_i^t \Delta_{i0}]$, which is an impossible event according to the model. This problem is observed to be persistent in the case of experiment 3, subjects 5, 6, 11, and 12. The time series for subject 11 as shown in Figure 10 demonstrates the phenomenon beginning with period 17 and remaining periods. In other cases this is mainly a short-term phenomenon which is corrected in one or two periods. This second type of inconsistency is not a possibility under Model 4 and this may be considered a point in favor of Model 4 in a choice among the two models.

Another advantage of Model 4 over Model 3 concerns the problem of observed values of the parameter ρ. Although we do not have estimates for the standard deviations of the estimated parameters for Models 3 and 4 to check whether the observed parameters are *statistically* greater than 1, mostly the ρ estimates are either less than or close to 1 (grid search for this parameter is performed in the range [0,2.0]). In terms of the number of cases with ρ estimates exceeding unity, Model 4 is less problematic than Model 3. We have twenty-three cases with ρ's over unity in Model 4, whereas in Model 3 thirty-six estimates are above 1. This may be considered another point in favor of Model 4 against Model 3 in that ρ's exceeding unity imply an individual assigning positive probability to an impossible event persistently.



(a) Model 3

(b) Model 4

FIGURE 9

Figure 9: Actual bids and predicted bids according to Model 3 and Model 4, and bounds for expectation of $Y_i(t)$ for subject 4 in experiment 5.

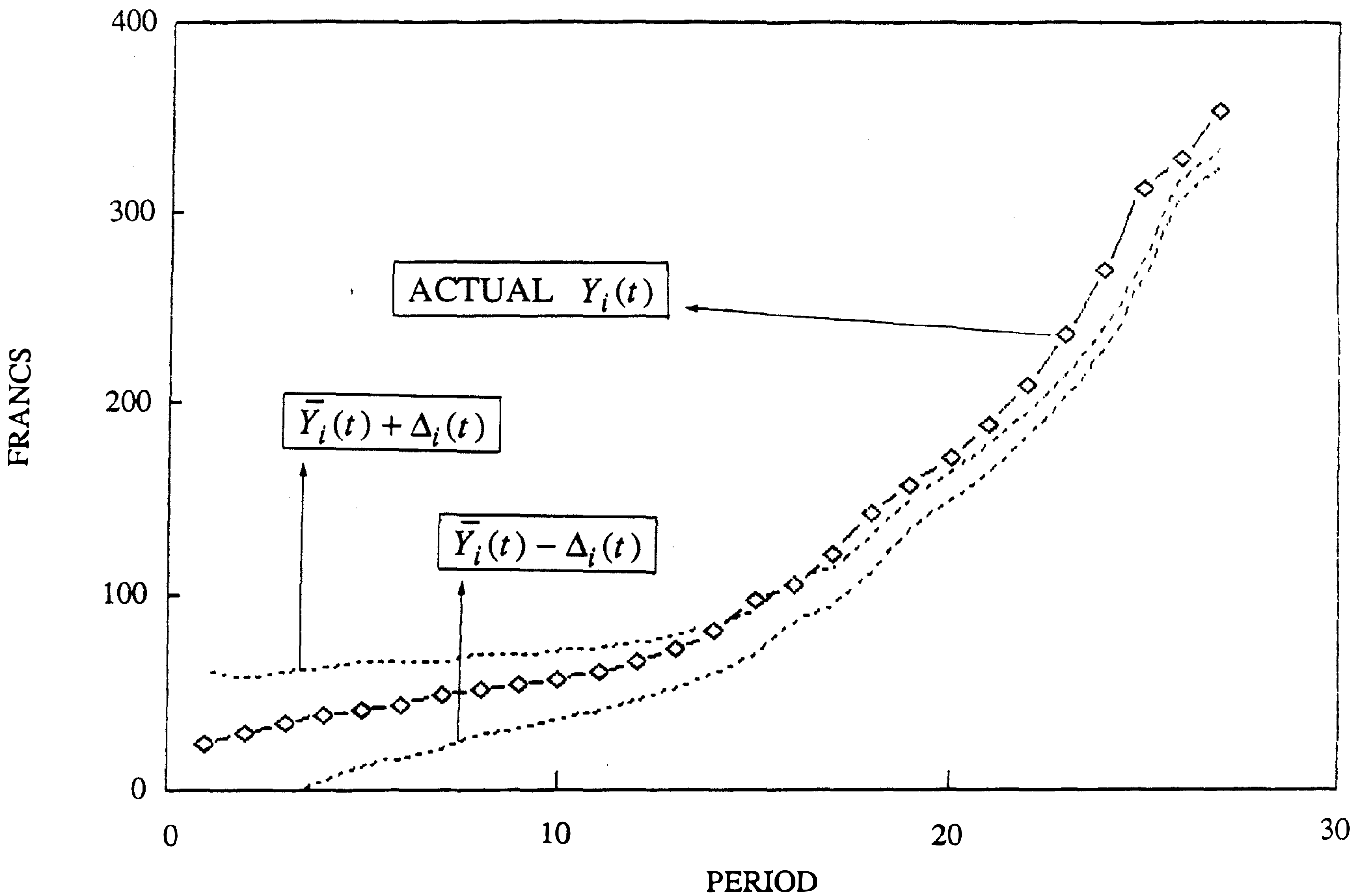


FIGURE 10

Figure 10: Bounds of expectations of $Y_i(t)$ (second lowest bid among i's rivals) according to model 3 and the realized value of $Y_i(t)$ for subject 11 in experiment 3.

SECTION VI. CONCLUDING REMARKS

Prior to gathering any experimental data when considering the zero-out auction as a solution to the Port's problem, two radically different classes of models existed in the deliberations. The first class involved models that are unconstrained by principles of game theory at either the system level or at the individual level of analysis. One model in this first class is the simple (myopic) demand and supply model in which the potential revenue returns are not reflected in the bidding decisions of participants. Another model in this class is what we termed the general linear adaptive model which postulates simple mechanical markup rules used by participants. The second class consists of models that are based on principles of game theory to varying degrees.

The data demonstrate that game-theoretic principles are relevant. After many periods of stationary parameters, the zero-out auctions studied here converge near the Distinguished Nash equilibrium of the static game-theoretic model (Conclusion 1). This result is consistent with a long history of experimental work that demonstrates convergence to static Nash equilibria. Thus the data led us to reject both the simple demand and supply model (Conclusion 2) and what we call Model 1, the general linear adaptive model, in favor of models in the second class with game-theoretic content (Conclusion 8 and Conclusion 9). Clearly the full set of the principles which led to the static Nash equilibrium as defined in Section III are *not* relevant because the equilibrium is not immediately attained. Some sort of convergence is observed. The problem thereby posed is to determine which principles should be retained and how they should be implemented for statistical purposes.

Two different approaches were used to develop models that utilize game-theoretic principles. Both approaches yield models that converge to the Distinguished Nash equilibrium so both have *a priori* consistency with important stylized facts (Proposition 2 and Proposition 3). The first was the traditional approach in which the principles governing system behavior were imposed. This approach is captured in Model 2, the strategic linear adaptive model, in which the equilibrium conditions are imposed at every period. Properties of the system equilibrium conditions are used to define the decision rules of specific individuals. In the second approach the system equilibrium conditions are dropped but optimization conditions are imposed at the individual level of analysis. Both approaches permit aspects of learning, or updating from experience.

Analysis of the data led to a rejection of the traditional approach (Conclusion 7) in favor of the decision-theoretic approach of Models 3 and 4 (Conclusion 9). Principles of individual optimization at every instant appear to be natural but the imposed system equilibrium conditions when modeling the dynamic adjustment process do not. The distinction between models 3 and 4 led to a further refinement of the principles of individual decisions. The implicit "irrationality" that occurs when equilibrium conditions are dropped appear in the expectations formation of individuals. Model 3 assumes that expectations reflect only learning in the sense of updating from prior experience. Model 4 goes further and allows the individual to be forward looking to a degree reminiscent of the absence of surprises dictated by a full game-theoretic specification. Individual beliefs about other's actions are on average correct in Model 4. Model 4 seems to best fit the data and it is encouraging that this different approach to dynamics has resulted in models that are such good competitors within sample and one which is in fact marginally better in terms of out-of-sample predictions (Conclusion 9).

We think that the evolution of thinking that resulted in the series of models in Section V demonstrates the importance of structural modeling in forecasting, and the unique role of *experimental data* from an econometric point of view. Not only are the econometric tools useful in giving direction to the experimental method (Conclusion 3) but the experimental data provide a special opportunity to study econometric methods. To make this last point clearer, let us assume that the data we have were field data, and we were interested in testing the specification given in Section III.A, i.e., linear adaptive model, and that we only had such information about the mechanism by which the data were generated that might exist in the field case. It is clear that we would be restricted to a subset of the tests performed in Section V.A, namely, to tests on the specification of error terms (Conclusion 4)and on the *signs* of some parameters, since we would not have strong theoretical restrictions on the values of some parameters. Under such conditions it is likely that we would have failed to reject the linear adaptive model; results for experiment 1 are a case in point. The fact that the data are obtained in an experimental setting with *known* values for some parameters—capacities and values—enables us to test the model by comparing the coefficients obtained when the model is imposed on the data to the known parameter values. Observe that for the first three experiments it is mainly these additional restrictions which cause rejection of the model which performed rather well in terms of standard tests on the error terms (Conclusion 6). This we think highlights a unique role for experiments in generating data on which the validity of an econometric model can be more thoroughly tested.[9] Notice also that these restrictions allow us to concentrate on other characteristics of an otherwise intractable model, e.g., equation (20) in Section V.B.

We close with observations about the Port's problem which is considerably more complicated than the problems we have been able to address. Two issues seem to be of central relevance to those interested in the applied problem as seen by the Port. First, inflation of bids should be expected. The amount of inflation could be on the order of the Nash equilibrium if one exists. Secondly, many features of the markets studied here cannot be relied upon in the field case of the Port and the consequences of these features on existence of equilibria and are unknown. In particular, the exogenous correlation between values and capacities is important for existence and it is also likely to be important for efficiency. The absence of an aftermarket is important to existence. A restriction on the number of slots a carrier can buy is also important for existence of the static equilibrium. Of course the values of V_i and X_i are almost certainly to be endogenous rather than constants as carriers use price concessions to maintain high levels of capacity. In view of the complexity of the applied problem, the advances offered here are certainly modest. However, several of the empirical questions are well within the grasp of current experimental techniques and the development of an appropriate model should be along the lines of Model 3 and Model 4.

APPENDIX A

NOTATION:

V_i = redemption value of the ith player ($i = 1, \ldots, 12$)
B_{it} = bid of the ith player at time t
Z_i = rebate factor for player i
R_t = total revenue at time t
R_{it}^* = expectation of R for the ith player

ASSUMPTIONS:

A1. Bid Equation

(a) $B_{it} = \alpha_i + \beta_i R_{it}^* + u_{it} \quad t = 1, \ldots, T; \; i = 1, \ldots, 12$

(b) for any i $u_{it} \sim iid\ N(0,\sigma_i^2)$

A2. Expectation Formation

(a) $R_{it}^* = \delta_i R_{t-1} + (1-\delta_i)R_{i,t-1}^* \quad t = 2, \ldots, T; \; i = 1, \ldots, 12$

(b) R_{i1}^* treated as an unknown constant.

ESTIMATION:

The unknown parameters are $(\alpha_i, \beta_i, \delta_i, R_{i1}^*, \sigma_i^2) \quad i = 1, \ldots, 12.$

A2.a. gives for the ith player:

$$R_{it}^* = \delta_i [R_{t-1} + (1-\delta_i)R_{t-2} + \cdots + (1-\delta_i)^{t-2}R_1] + (1-\delta_i)^{t-1}R_{i1}^*, \quad t \geq 2.$$

So the model for the ith player is

$$B_{i1} = \alpha_i + \beta_i R_{i1}^* + u_{i1}$$

$$B_{it} = \alpha_i + \beta_i \delta_i [R_{t-1} + (1-\delta_i)R_{t-2} + \cdots + (1-\delta_i)^{t-2}R_1] + (1-\delta_i)^{t-1}R_{i1}^* + u_{it}, \quad t > 2.$$

Define $\mu_i = \alpha_i + \beta_i R_{i1}^*$, $\lambda_i = (1-\delta_i)$, $\alpha_i^* = \alpha_i \delta_i$, $\beta_i^* = \beta_i \delta_i$. So the new parameters are $(\alpha_i^*, \beta_i^*, \lambda_i, \mu_i, \sigma_i^2)$. The model becomes:

$$B_{i1} = \mu_i + u_{i1}$$

$$B_{it} = \mu_i \lambda_i^{t-1} + \alpha_i^*[1 + \lambda_i + \cdots + \lambda_i^{t-1}] + \beta_i^*[R_{t-1} + \lambda_i R_{t-2} + \cdots + \lambda_i^{t-2}R_1] + u_{it}, \quad t > 2.$$

The joint log-likelihood of $(B_{i2}, \ldots, B_{iT})$ is:

$$L_i = k - (T/2)\log \sigma_i^2 - (1/2\sigma_i^2)(B_{i1} - \mu_i) - (1/2\sigma_i^2)\sum_{t=2}^{T}(B_{it} - \mu_i\lambda_i^{t-1} - \alpha_i^* 1_t(\lambda_i) - \beta_i^* R_t(\lambda_i))^2$$

where

$$1_t(\lambda_i) = 1 + \lambda_i + \lambda_i^2 + \cdots + \lambda_i^{t-1}$$

$$R_t(\lambda_i) = R_{t-1} + \lambda_i R_{t-2} + \cdots + \lambda_i^{t-2} R_i$$

k = constant.

Maximization of L_i wrt $(\alpha_i^*, \beta_i^*, \mu_i, \lambda_i, \sigma_i^2)$ gives the maximum likelihood estimator which is strongly consistent and asymptotically efficient for $(\alpha_i^*, \beta_i^*, \lambda_i, \sigma_i^2)$ but not consistent for μ_i (see e.g., Judge et al. (1985) p. 383) given the correct specification of the model.

An asymptotically equivalent estimator is obtained by omitting the first observation. This latter estimator is easier to compute and is obtained by maximizing the marginal log-likelihood for $(B_{i2}, \ldots, B_{it})$:

$$\tilde{L}_i = k - (1/2)(T-1)\log\sigma_i^2 - (1/2\,\sigma_i^2)\sum_{t=2}^{T}(B_{it} - \mu_i\lambda_i^{t-1} - \alpha_i^* 1_t(\lambda_i) - \beta_i^* R_t(\lambda_i))^2$$

(see Klein (1958), Zellner and Geisel (1970)).

Let

$$X(\lambda_i) = \begin{bmatrix} \lambda_i & 1 & R_1 \\ \lambda_i^2 & 1+\lambda_i & R_2 + \lambda_i R_1 \\ \vdots & \vdots & \\ \lambda_i^{T-1} & 1 + \lambda_i + \cdots + \lambda_i^{T-2} & R_{T-1} + \lambda_i R_{T-2} + \cdots + \lambda_i^{T-2} R_1 \end{bmatrix}$$

$$B_i' = (B_{i2}, \ldots, B_{iT})'.$$

The (marginal) ML estimator is given by:

$$\begin{bmatrix} \hat{\mu}_i \\ \hat{\alpha}_i^* \\ \hat{\beta}_i^* \end{bmatrix} = (\hat{X}_i' \hat{X}_i)^{-1} \hat{X}_i' B_i \quad \text{where } \hat{X}_i = X(\hat{\lambda}_i)$$

$$\hat{\sigma}_i^2 = [1/(T-1)]\, SSR_i(\hat{\lambda}_i) \equiv [1/(T-1)]\, B_i'[I_{T-1} - \hat{X}_i(\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i']\, B_i$$

and $\hat{\lambda}_i$ minimizes $SSR_i(\lambda_i)$. This value $\hat{\lambda}_i$ is obtained by a grid-search over $(-1,1)$ for λ_i.

To obtain the approximate covariance matrix of the ML estimator $(\hat{\alpha}_i^*, \hat{\beta}_i^*, \hat{\lambda}_i)$, we define

$$\gamma_i = \begin{bmatrix} \alpha_i^* \\ \beta_i^* \end{bmatrix}; \quad \theta_i = \begin{bmatrix} \mu_i \\ \gamma_i \end{bmatrix};$$

$$x_t'(\lambda_i) = (\lambda_i^{t-1}, 1_t(\lambda_i), R_t(\lambda_i)), \quad (t\text{th row of } X(\lambda_i))$$

$$w'(\lambda_i) = (1_t(\lambda_i), R_t(\lambda_i))$$

so that $W(\lambda_i)$ is obtained from $X(\lambda_i)$ by deleting the first column. The likelihood for the tth observation ($t \geq 2$) is:

$$\ell_{it} = \text{constant} - (1/2)\log \sigma_i^2 - (1/2\sigma_i^2)\,[B_{it} - x_t'(\lambda_i)\theta_i]^2$$

$$\frac{\partial \ell_{it}}{\partial \gamma_i} = (1/\sigma_i^2)[B_{it} - x_t'(\lambda_i)\theta_i]\, w_t(\lambda_i)$$

$$\frac{\partial \ell_{it}}{\partial \lambda_i} = (1/\sigma_i^2)[B_{it} - x_t'(\lambda_i)\theta_i] dx_t'(\lambda_i)\theta_i$$

where

$$dx_t'(\lambda_i) = \begin{cases} [1,0,0] & \text{for } t = 2 \\ [(t-1)\lambda_i^{t-1},\ 1 + 2\lambda_i + \cdots + (t-2)\lambda_i^{t-3},\ R_{t-2} + 2\lambda_i R_{t-3} + \cdots + (t-2)\lambda_i^{t-3} R_1] & \text{for } t > 2. \end{cases}$$

$$\frac{\partial^2 \ell_{it}}{\partial \gamma_i \partial \gamma_i'} = -(1/\sigma_i^2) w_t(\lambda_i) w_t'(\lambda_i)$$

$$\frac{\partial^2 \ell_{it}}{\partial \gamma_i \partial \gamma_i} = -(1/\sigma_i^2) dx_t'(\lambda_i)\theta_i w_t(\lambda_i) + (1/\sigma_i^2)[B_{it} - x_t'(\lambda_i)\theta_i] dw_t(\lambda_i)$$

where $dw_t'(\lambda_i)$ is obtained from $dx_t'(\lambda_i)$ by deleting the first column.

$$\frac{\partial^2 \ell_{it}}{\partial \lambda_i \partial \lambda_i} = -(1/\sigma_i^2)[dx_t'(\lambda_i)\theta_i]^2 - (1/\sigma_i^2) x_i'(\lambda_i)\theta_i\, d^2 x_t'(\lambda_i)\theta_i$$

where

$$d^2 x_t'(\lambda_i) = \begin{cases} [0,0,0] & \text{for } t = 2 \\ [t-1,1,0] & \text{for } t = 3 \\ [(t-1)(t-2)\lambda_i^{t-3},\ 2 + 6\lambda_i + \cdots + (t-2)(t-3)\lambda_i^{t-4},\ 2R_{t-3} + 6\lambda_i R_{t-4} + \cdots + (t-2)(t-3)\lambda_i^{t-4} R_1 & \\ & \text{for } t > 3\,. \end{cases}$$

Define

$$\hat{D} = \sum_{t=2}^{T} \begin{bmatrix} \dfrac{\partial \hat{\ell}_{it}}{\partial \gamma_i} \\ \dfrac{\partial \hat{\ell}_{it}}{\partial \lambda_i} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \hat{\ell}_{it}}{\partial \gamma_i'}, & \dfrac{\partial \hat{\ell}_{it}}{\partial \lambda_i} \end{bmatrix}$$

$$\hat{D} = (1/\hat{\sigma}_i^4) \sum_{t=2}^{T} \hat{e}_{it}^2 \begin{bmatrix} w_t(\hat{\lambda}_i) \\ dx_t'(\hat{\lambda}_i)\hat{\theta}_i \end{bmatrix} [w_t'(\hat{\lambda}_i) \cdot dx_t'(\hat{\lambda}_i)\hat{\theta}_i]$$

where $\hat{e}_{it} = B_{it} - x_t'(\hat{\lambda}_i)\hat{\theta}_i$ is the OLS residuals. Define

$$d\hat{X}_i = \begin{bmatrix} \vdots \\ dx_t'(\hat{\lambda}_i) \\ \vdots \end{bmatrix}$$

$$\hat{W}_i = \begin{bmatrix} \vdots \\ w_t'(\hat{\lambda}_i) \\ \vdots \end{bmatrix}$$

$$\hat{E}_i^2 = diag\,[\hat{e}_{i2}^2, \ldots, \hat{e}_{iT}^2].$$

Then

$$\hat{D} = (1/\hat{\sigma}_i^4) \begin{bmatrix} \hat{W}_i' \\ \hline \hat{\theta}_i' d\hat{X}_i \end{bmatrix} \hat{E}_i^2 [\hat{W}_i \mid d\hat{X}_i \hat{\theta}_i]$$

Define

$$\hat{A} = \sum_{t=2}^{T} \begin{bmatrix} \dfrac{\partial^2 \hat{\ell}_i}{\partial \gamma_i \partial \gamma_i'} & \dfrac{\partial^2 \hat{\ell}_i}{\partial \gamma_i \partial \lambda_i} \\ \dfrac{\partial^2 \hat{\ell}_i}{\partial \lambda_i \partial \gamma_i'} & \dfrac{\partial^2 \hat{\ell}_i}{\partial \lambda_i \partial \lambda_i} \end{bmatrix}$$

$$\hat{A} = -(1/\hat{\sigma}_i^2) \begin{bmatrix} \hat{W}_i' \\ \hline \hat{\theta}_i' d\hat{X}_i' \end{bmatrix} [\hat{W}_i \mid d\hat{X}_i \hat{\theta}_i] + (1/\hat{\sigma}_i^2) \left[\begin{array}{c|c} 0 & d\hat{W}_i' \hat{e}_i \\ \hline \hat{e}_i' d\hat{W}_i & 0 \end{array}\right]$$

where $d\hat{W}_i$ is obtained from $d\hat{X}_i$ by deleting the first column.

Then the White (1981) robust approximate covariance matrix of the ML estimator $(\hat{\alpha}_i^*, \hat{\beta}_i^*, \hat{\lambda}_i)$ is

$$Ap.\ Var. \begin{bmatrix} \hat{\alpha}_i^* \\ \hat{\beta}_i^* \\ \hat{\lambda}_i \end{bmatrix} = \hat{A}^{-1} \hat{D}\, \hat{A}^{-1}.$$

If the model is correctly specified then one can use, instead, either

$$Ap.\ Var.\begin{bmatrix}\hat{\alpha}_i^*\\ \hat{\beta}_i^*\\ \hat{\lambda}_i\end{bmatrix} = -\hat{A}^{-1} \quad or$$

$$Ap.\ Var.\begin{bmatrix}\hat{\alpha}_i^*\\ \hat{\beta}_i^*\\ \hat{\lambda}_i\end{bmatrix} = \hat{D}^{-1}.$$

The ML estimator of the initial parameters (α_i, β_i) is obtained by

$$\hat{\alpha}_i = \hat{\alpha}_i^*/(1-\hat{\lambda}_i)$$
$$\hat{\beta}_i = \hat{\beta}_i^*/(1-\hat{\lambda}_i)$$

The approximate covariance matrix of $(\hat{\alpha}_i, \hat{\beta}_i)$ is given by

$$Ap.\ Var.\begin{bmatrix}\hat{\alpha}_i^*\\ \hat{\beta}_i^*\end{bmatrix} = \hat{P}'\ Ap.\ Var.\begin{bmatrix}\hat{\alpha}_i^*\\ \hat{\beta}_i^*\\ \hat{\lambda}_i\end{bmatrix}\hat{P}$$

where $Ap.\ Var.\begin{bmatrix}\hat{\alpha}_i^*\\ \hat{\beta}_i^*\\ \hat{\lambda}_i\end{bmatrix}$ is given above and

$$\hat{P}' = \begin{bmatrix}1/(1-\hat{\lambda}_i) & 0 & \hat{\alpha}_i^*/(1-\hat{\lambda}_i)^2\\ 0 & 1/(1-\hat{\lambda}_i) & \hat{\beta}_i^*/(1-\hat{\lambda}_i)^2\end{bmatrix}.$$

SPECIFICATION TESTING:
For any individual i, we want to test that the individual's bid B_{it} deviates from the model prediction $\alpha_i + \beta_i R_{it}^*$ by an error u_{it} which is uncorrelated with the individual's expectation R_{it}^* as claimed by assumption A1.a: $B_{it} = \alpha_i + \beta_i R_{it}^* + u_{it}$.

However, R_{it}^* and u_{it} are both unobserved. In addition, the estimated expectation $\hat{R}_{it}^* = \hat{\delta}_i[R_{t-1} + (1-\hat{\delta}_i)R_{t-2} + \cdots + (1-\hat{\delta}_i)^{t-2}R_1] + (1-\hat{\delta}_i)^{t-2}\hat{R}_{i1}^*$ is by construction of the ML estimates orthogonal to the estimated error $\hat{u}_{it} = B_{it} - \hat{\mu}_i\hat{\lambda}_i^{t-1} - \hat{\alpha}_i^* 1_t(\hat{\lambda}_i) - \hat{\beta}_i R_t(\hat{\lambda}_i)$.

We shall use a Hausman (1978) test which requires an additional estimator of θ. This estimator is an instrumental variables (IV) estimator (see, e.g., Liviatan (1963), Judge et al. (1985, pp. 390-391). For $t \geq 2$, we have

$$B_{it} = \alpha_i + \beta_i R_{it}^* + u_{it}$$

$$B_{i,t-1} = \alpha_i + \beta_i R_{i,t-1}^* + u_{i,t-1} .$$

Multiplying the second equation by λ_i and subtracting it from the first gives, using A2.a,

$$B_{it} = \alpha_i^* + \beta_i^* R_{t-1} + \lambda_i B_{i,t-1} + (u_{it} - \lambda_i u_{i,t-1}) \quad \text{for } t \geq 2.$$

B_{it} is correlated with $(u_{it} - \lambda_i u_{i,t-1})$. But note that $R_{t-1} = \sum_{j \varepsilon W_{t-1}} B_{j,t-1}$ may also be correlated with $(u_{it} - \lambda_i u_{i,t-1})$. Thus, we use three instrumental variates which are $(1, R_{t-2}, R_{t-3})$. Deleting the first 2 observations, IV estimator of $(\alpha_i^*, \beta_i^*, \lambda_i)$ is

$$\tilde{\theta}_i = \begin{bmatrix} \tilde{\sigma}_i^* \\ \tilde{\beta}_i^* \\ \tilde{\lambda}_i^* \end{bmatrix} = (R' X_i)^{-1} R' B_i$$

where

$$B_i = (B_{i3}, \ldots, B_{iT})$$

$$R = \begin{bmatrix} 1 & R_2 & R_1 \\ 1 & R_3 & R_2 \\ \vdots & \vdots & \vdots \\ 1 & R_{T-1} & R_{T-2} \end{bmatrix} \qquad X_i = \begin{bmatrix} 1 & R_2 & B_{i,2} \\ 1 & R_3 & B_{i,3} \\ \vdots & \vdots & \vdots \\ 1 & R_{T-1} & B_{i,T-1} \end{bmatrix}$$

Approximate covariance matrix of the IV estimator is

$$Ap.\ Var. \begin{bmatrix} \tilde{\alpha}_i^* \\ \tilde{\beta}_i^* \\ \tilde{\lambda}_i^* \end{bmatrix} = \tilde{s}_i^2 (R' X_i)^{-1} R' \hat{\Psi}_i R (X_i' Z)^{-1}$$

where

$$\tilde{s}_i^2 = [1/(1 + \tilde{\lambda}_i^2)][B_i - X_i \tilde{\theta}_i]'[B_i - X_i \tilde{\theta}_i]$$

$$\hat{\Psi}_i = \begin{bmatrix} 1+\tilde{\lambda}_i^2 & -\tilde{\lambda}_i & 0 & \cdots & 0 \\ -\tilde{\lambda}_i & 1+\tilde{\lambda}_i^2 & -\tilde{\lambda}_i & \cdots & 0 \\ 0 & -\tilde{\lambda}_i & 1+\tilde{\lambda}_i^2 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 1+\tilde{\lambda}_i^2 \end{bmatrix}$$

(see Liviatan (1963)).

Then IV estimator of α_i and β_i are

$$\tilde{\alpha}_i = \alpha_i^*/(1-\tilde{\lambda}_i)$$

$$\tilde{\beta}_i = \beta_i^*/(1-\tilde{\lambda}_i)$$

The approximate covariance matrix of $(\tilde{\alpha}_i, \tilde{\beta}_i)$ is given by

$$Ap.\ Var. \begin{bmatrix} \tilde{\alpha}_i \\ \tilde{\beta}_i \end{bmatrix} = \tilde{P}_i'\ Ap.\ Var.\ \tilde{\theta}\ \ \tilde{P}_i$$

where $\tilde{P}_i$ is defined as $\hat{P}_i$ above with $\tilde{\lambda}_i$ replacing $\hat{\lambda}_i$. The Hausman (1978) test for testing the hypothesis that the expectation R_{it}^* is uncorrelated with the residual, u_{it}, is

$$H_i = (\tilde{\beta}_i - \hat{\beta}_i)^2/(Ap.\ Var.\ \tilde{\beta}_i - Ap.\ Var.\ \hat{\beta}_i)$$

where Ap. Var. $\hat{\beta}_i$ is the approximate variance of the ML estimator $\hat{\beta}_i$ of β_i under correct specification.

$H_i \xrightarrow{D} \chi^2_{(1)}$ under the null hypothesis of no correlation.

For any individual i, we also want to test that the best (in the mean squared error sense) linear predictor of the individual i's bid B_{it} is $\alpha_i + \beta_i R_{it}^*$. This is done by testing the u_{it} are white noise as asserted by A1.b: $u_{it} \sim iid\ N(0, \sigma_i^2)$. However, the expectation R_{it}^* may contain lagged endogenous variables since $R_t = \sum_{j \varepsilon W_t} B_{jt}$, so that the usual Durbin-Watson (1950) statistic for testing against u_{it} being an AR(1) is inappropriate. Durbin (1970)'s h-statistic for testing against u_{it} being an AR(1) process cannot also be applied.

We use Godfrey (1978) tests which can easily seem to be applicable even when the model is nonlinear. To test $H_0 : u_{it} \sim iid\ N(0,)$ against $H_A^P : u_{it} \sim AR(p)$ or $u_{it} \sim MA(p)$. Let $\hat{u}_i = B_i - X(\hat{\lambda}_i)\hat{\theta}_i$ (the vector of estimated residuals),

$$\hat{u}_{ik}' = (\underbrace{0, 0, \ldots, 0}_{k\ zeros}, u_2, \ldots, u_{T-k})'$$

and $\hat{U}_{iP} = [\hat{u}_{i1}, \ldots, \hat{u}_{iP}]$.

The ML estimator of the first p auto-correlation coefficients of u_{it} are given by $(\rho_1, \ldots, \rho_P)' = (1/T\hat{\sigma}_i^2)\hat{U}_{iP}'\hat{u}_i$ where $\hat{\sigma}_i^2$ is the ML estimate of σ_i^2 given above.

The statistic for testing against H_A^P is $G_i^P = (T-1)R_{iP}^2$ where R_{iP}^2 is the R^2 of the regression of $\hat{u}_i$ on $[X(\hat{\lambda}_i);\ \hat{U}_{iP}]$.

$$G_i^P \xrightarrow{D} \chi^2_{(p)} \text{ under } H_0.$$

We test H_0 against H_A^1.

APPENDIX B

INSTRUCTIONS

This is an experiment in the economics of market decision-making. Various research foundations have provided funds for the conduct of this research. The instructions are simple, and if you follow them carefully and make good decisions you might earn a considerable amount of money which will be paid to you in cash after the experiment. Please check your folder for the contents listed on the chalkboard. The information in the folder is your own private information. You are not to reveal this information to anyone.

In this experiment we are going to conduct two separate markets called Blue and Yellow. The market in which you will participate has already been determined. Those with blue forms and with blue printed by their identification number on the folder will participate in the Blue Market. Those with yellow forms will participate in the Yellow market. The currency to be used in the experiment is francs. Each franc is worth _____ dollars to you.

A fixed number of units will be sold in each market to the highest bidders. Each individual can purchase at most one unit. Your earnings are computed as follows. If you are awarded a unit from the auction you receive a redemption value for the acquired unit as shown in row 1 of your information sheet. In addition, if you acquire a unit in the auction you receive a proportion of the revenue from *both* auctions as a rebate. In addition to any earnings from the auction, you are paid a capital payment each period. The amount of the capital payment is listed in row 6 of your information sheet.

If you acquire a unit

earnings = redemption value + rebate – your bid + capital payment

If you do not acquire a unit

earnings = capital payment

The blanks on the Record sheet will help you record your profits. Enter your bid on line 4 when your bid is submitted. *If you acquire a unit* during the first period the redemption value should be recorded on row 1 at the time of purchase. You should then add the rebate and subtract what you paid for the unit (your bid) as directed on rows 3 and 4. At the end of the period record the total of profits and capital payment on the last row on the page. Subsequent periods should be recorded similarly.

Computation of the rebate

Rebates of winning bidders will be announced after each period. However, we show you how the number is calculated so those who did not win can determine what their rebate might have been.

On the information sheet you will see a capacity number assigned to each bidder. These numbers are used in the following two formulas:

$$\text{Your rebate} = \left[\text{Your rebate factor}\right] \cdot \left[\begin{array}{c}\text{Total sales revenue}\\ \text{in both markets}\end{array}\right]$$

$$\text{Your rebate} = \left[\frac{\text{Your capacity}}{\begin{array}{c}\text{all winning bidders}\\ \text{in both markets}\end{array}}\right] \cdot \left[\begin{array}{c}\text{Sales revenue}\\ \text{in Blue market}\end{array} + \begin{array}{c}\text{Sales revenue}\\ \text{in Yellow market}\end{array}\right]$$

Bidding

The market for this commodity will be organized as follows: we open the markets for each trading period by announcing the total number of items for sale during that period. Each of you as a buyer purchase units by submitting a bid, on a bid form, which may be accepted or rejected. The bid form will include your buyer number, the period number, and your bid for the unit.

Bids are accepted or rejected each period as follows: the bids will be collected from all buyers and ordered from the highest bid to the lowest bid. If n units are to be sold then the highest n bids will be accepted. For example, with 8 units offered for sale, the highest 8 bids will be accepted, and the lower bids will be rejected. In the case of ties at the lowest accepted bid, random numbers will be used to determine which of the bids will be accepted. In each market the highest bid, the lowest accepted bid and the highest rejected bid will then be announced. The winners and the amount of each rebate will be announced. Each buyer will then fill in his record of purchases and earnings for that period for the accepted bids.

TEST

1. Suppose that 3 units are available in each market, and the bids are as given below.

	Blue					Yellow				
Individual	A	B	C	D	E	F	G	H	I	J
Bid	30	20	10	5	4	20	10	5	3	2
Capacities	1	3	6	7	9	10	8	5	4	2

   a. Determime the winners in Blue and in Yellow. ____________________

   b. Determime total revenue. ____________________

   c. Determine the rebate factor for B: ____________________

2. Suppose your bid is 60, total revenue is 500, and your rebate factor is 10%.

   a. Calculate your rebate. ____________________

   b. If your value is 100, what is your profit? ____________________

R E C O R D   S H E E T

Buyer No. ______

CAPACITY = ______

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1 Redemption Value															
2 Rebate															
3 Net Income (row 1+row 2)															
4 Bid															
5 Total per period (row 3-row 4)															
6 Capital Payment															
7 Earnings															

NAME ____

SOCIAL SECURITY NO. ____________________________

R E C O R D   S H E E T

Buyer No. ______   (continued)   CAPACITY = ______

Period	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1 Redemption Value															
2 Rebate															
3 Net Income (row 1+row 2)															
4 Bid															
5 Total per period (row 3-row 4)															
6 Capital Payment															
7 Earnings															

NAME ________________________________

SOCIAL SECURITY NO. ____________________

I N F O R M A T I O N   S H E E T

Your redemption value

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Redemption value															

Capacities of all people in the __________ market

Person	1	2	3	4	5	6	7	8	9	10
Capacity										

FOOTNOTES

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1. What would be the implications for carriers that use New York airports as hubs? Does the process impact differently between small and large carriers or between carriers that operate during several times of day as opposed to only one time of day? Would carriers that operate at several different New York airports be advantaged or disadvantaged? Would entry, airfares, etc. be affected?

2. In what follows we use the same notation for a set and its cardinality.

3. The other extra marginal bidder can bid anything as long as it is less than or equal to $\underline{B}_k^*$.

4. In estimating (13) we lose two observations in experiment 2 market 2 and experiment 3 market 2 since the change in the minimum accepted bid is negative.

5. We neglect the possible correlation between u_{it} and u_{jt}, $i \neq j$. This leads to inefficient but not inconsistent estimates provided u_{it} and R_{it}^* are uncorrelated. This latter condition is satisfied if u_{it} is uncorrelated over time and R_{it}^* depends only on the past, as assumed.

6. For two cases we get negative values for the Hausman statistic because of nonconvergence of the maximum likelihood procedure.

7. The standard deviation estimates in the tests on coefficients are White (1982) robust estimates.

8. Models 3 and 4 have the same *RMSE* in one case and the same Theil's U in two cases. Models 1 and 2 have the same Theil's U in one case. These ties cause row totals in the comparison table to add up to more than seventy-two in some comparisons.

9. For similar use of field experiments in evaluating econometric models, see R. J. LaLonde (1986).

REFERENCES

Cagan, P. "The Monetary Dynamics of Hyper-Inflation." In *Studies in the Quantity of Money,* edited by H. Friedman. Chicago: University of Chicago Press, 1956.

Daniels, Bryan P. and Plott, Charles R. "Inflation and Expectations in Experimental Markets." Social Science Working Paper no. 634. Pasadena: California Institute of Technology, May 1987. Forthcoming in *Lecture Note Series,* edited by Reinhard Tietz. Berlin-Heidelberg-New York: Springer.

Durbin, J. "Testing for Serial Correlation in Least Squares Regression When Some of the Regressors Are Lagged Dependent Variables." *Econometrica* 38 (1970): 410-421.

Durbin, J. and Watson, G. S. "Testing for Serial Correlation in Least Squares Regression J." *Biometrika* 37 (1950): 409-428.

Easley, David and Ledyard, John. "Theories of Price Formation and Exchange in Double Oral Auctions." Social Science Working Paper no. 611. Pasadena: California Institute of Technology, April 1986.

Flight Transportation Associates, Inc. "Report to the Port Authority of New York and New Jersey." Mimeographed. New York, 1985.

Friedman, Daniel. "On the Efficiency of Experimental Double Auction Markets." *American Economic Review* 74 (March 1984): 60-72.

Godfrey, L. G. "Testing Against General Autoregressive and Moving Average Error Models When the Regressors Include Lagged Dependent Variables." *Econometrica* 46 (1978): 1293-1301.

Hausman, J. A. "Specification Tests in Econometrics." *Econometrica* 46 (1978): 1251-1271.

Judge, G. G.; Griffiths, W. E.; Hill, R. C.; Lee, T. *The Theory and Practice of Econometrics.* New York: Wiley, 1980.

Klein, L. R. "Estimation of Distributed Lags." *Econometrica* 26 (1958): 553-565.

LaLonde, R. J. "Evaluating the Econometric Evaluations of Training Programs with Experimental Data." *American Economic Review* 76 (1986): 604-620.

Liviatan, N. "Consistent Estimation of Distributed Lags." *International Economic Review* 4 (1963): 44-52.

Muth, J. F. "Optimal Properties of Exponentially Weighted Forecasts." *Journal of the American Statistical Association* 55 (1960): 299-305.

Nerlove, M. "Estimates of the Elasticities of Supply of Selected Agricultural Commodities." *Journal of Farm Economics* 38 (1956): 496-509.

Plott, C. R. "Industrial Organization Theory and Experimental Economics." *Journal of Economic Literature* 20 (1982): 1485-1527.

Smith, Vernon L. "Microeconomic Systems as an Experimental Science." *American Economic Review* 72 (1982): 923-955.

White, H. "Maximum Likelihood Estimation of Misspecified Models." *Econometrica* 50 (1982): 1-25.

White H. and Domowitz, I. "Non-Linear Regression with Dependent Observations." *Econometrica* 52 (1986): 143-161.

Williams, Arlington W. "The Formation of Price Forecasts in Experimental Markets." *Journal of Money, Credit, and Banking* 19 (1987): 1-18.

Wilson, Robert B. "Incentive Efficiency of Double Auctions." *Econometrica* 53 (1985): 1101-1116.

Wilson, Robert B. "Double Auctions." *Information, Incentives and Economic Mechanisms: Essays in Honor of Leonid Hurwicz*, edited by T. Groves, R. Radner, and S. Reiter. Minneapolis: University of Minnesota Press. Forthcoming (a).

Wilson, Robert B. "Equilibria of Bid-Ask Markets." *Arrow and the Ascent of Economic Theory: Essays in Honor of Kenneth J. Arrow*, edited by G. Feiwel. London: Macmillan Press Ltd. Forthcoming (b).

Zellner, A., and Geisel, M. "Analysis of Distributed Lag Models with Application to Consumption Function Estimation." *Econometrica* 38 (1970): 865-888.